

# Control of Wave Energy Devices

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# Outline

- 1 Wave Resource
- 2 WEC models
  - Linear models and Cummins' equation
  - Parameterising non-parametric hydrodynamic data
  - Nonlinear models and linearisation
  - Hydrodynamic models from data
- 3 WEC Control
- 4 Sensitivity Analysis
- 5  $F_{ex}$  Estimation and Forecasting
- 6 Conclusions

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## 1 Wave Resource

## 2 WEC models

- Linear models and Cummins' equation
- Parameterising non-parametric hydrodynamic data
- Nonlinear models and linearisation
- Hydrodynamic models from data

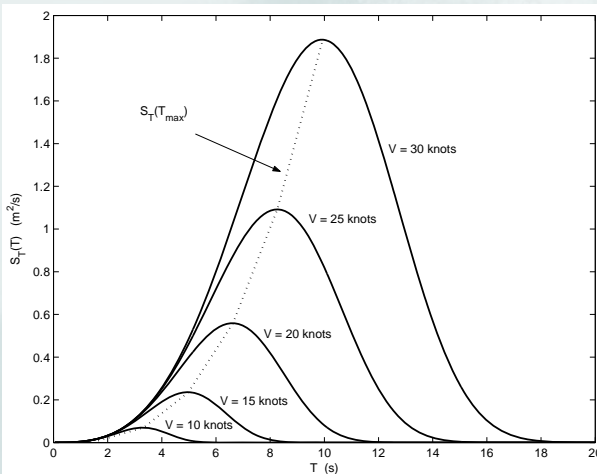
## 3 WEC Control

## 4 Sensitivity Analysis

## 5 $F_{ex}$ Estimation and Forecasting

## 6 Conclusions

# Typical P-M Amplitude Spectrum



Discrete frequencies with amplitudes from envelope and random phases used for simulation



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# Use of mathematical WEC models

- ① Assessment of power production
- ② Assessment of loading/forces under extreme sea conditions
- ③ Simulation of device and array motions,
  - Device geometry optimisation<sup>1 2</sup>
  - Array layout optimisation<sup>3</sup>
  - Evaluation of effectiveness of control strategies\*\*
- ④ For use as a basis for model-based control design\*\*

Currently, no 'super' model available<sup>4</sup>

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<sup>1</sup>Garcia Rosa, P.B. and Ringwood, J.V. On the sensitivity of optimal wave energy device geometry to the energy maximising control system. IEEE Trans. on Sustainable Energy, Vol.7, No.1, pp 419-426, 2016

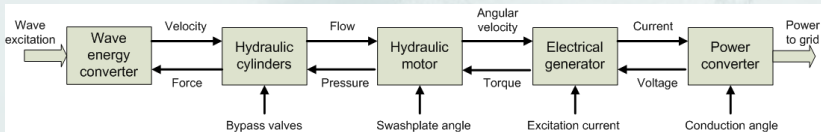
<sup>2</sup>Garcia Rosa, P.B. and Ringwood, J.V. Control-informed geometric optimisation of wave energy converters: The impact of device motion and force constraints, Energies, Vol.8, No.12, pp 13672-13687, 2015

<sup>3</sup>Garcia Rosa, P. B., Bacelli, G. and Ringwood, J.V.. Control-informed optimal layout for wave farms. IEEE Trans. on Sustainable Energy, Vol.6, No.2, pp 575-582, 2015

<sup>4</sup>Penalba, M. and Ringwood, J.V. A review of wave-to-wire models for wave energy converters, Energies, Vol.9, No.7, 506, 2016

# Wave-to-wire models

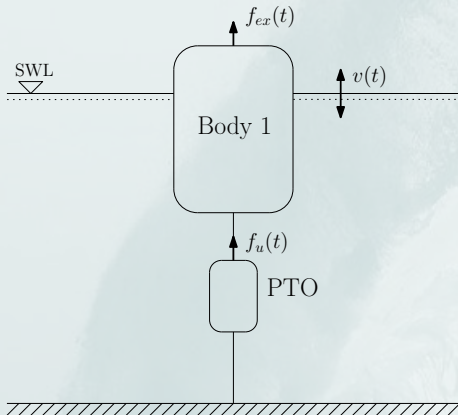
## Complete energy flow schematic:



- Multiple changes in form of power (hydrodynamic, mechanical, hydraulic, electrical...)
- Variety of ways to implement torque/force control on PTO
- Ideally, electrical grid should also be modelled
- Bond graphs provide a nice way to model different power/energy forms<sup>5</sup>

<sup>5</sup>Bacelli, G., Gilloteaux, J.-C. and Ringwood, J.V. State space model of a hydraulic power take off unit for wave energy conversion employing bondgraphs, Proc. World Renewable Energy Conference, Glasgow, 2008

# Simple heaving buoy



Useful converted power:

$$P_{PTO}(t) = f_u(t)v(t)$$

# Basic equation of motion

Following Newton's second law....

$$M\dot{v}(t) = f_m(t) + f_r(t) + f_d(t) + f_v(t) + f_b(t) + f_{ex}(t) + f_u(t) \tag{1}$$

where  $v(t)$  is the heaving velocity and  $M$  is the WEC mass and

- $f_m$  is the mooring force
- $f_r$  is the radiation force
- $f_d$  is the diffraction force
- $f_v$  is the viscous damping force
- $f_b$  is the buoyancy/gravity restoring force
- $f_{ex}$  is the wave excitation force
- $f_u$  is opposing PTO (control) force

# Linear approximation

With the assumptions of linear potential theory:

- ① Irrotational, incompressible and inviscid fluid,
- ② Small-body approximation (wave elevation constant across the whole body),
- ③ Small oscillations (constant wetted surface),

the following simplifying equations apply:

$$f_{ex} + f_d(t) = \int_{-\infty}^{+\infty} h_{ex}(\tau)\eta(t - \tau)d\tau \quad (2)$$

$$f_r(t) = - \int_0^t h_r(\tau)v(t - \tau)d\tau - m_{\infty}\dot{v}(t) \quad (3)$$

$$f_b(t) = -\rho g S_w \int_0^t v(\tau)d\tau = -K_b x(t) \quad (4)$$

$$f_v(t) = 0. \quad (5)$$

# Cummins equation

$$(M + m_{\infty})\dot{v}(t) + \int_0^{+\infty} h_r(\tau)v(t - \tau)d\tau + K_b x(t) = \underbrace{\int_{-\infty}^t h_{ex}(\tau)\eta(t - \tau)d\tau}_{F_{ex}(t)} + f_u(t) \quad (6)$$

$h_{ex}(t) [H_{ex}(\omega)]$  and  $h_r(t) [H_r(\omega)]$  are typically calculated numerically (non-parametric form) using boundary-element potential methods such as WAMIT, AQUAPLUS, NEMOH, AQWA or ACHIL3D

# Radiation damping approximations

Can replace the radiation damping convolution term in (3) by a closed form (finite order) equivalent:

- The integro-differential equation in (6) replaced by a higher-order differential equation, making analysis more straightforward,
- The resulting finite-order dynamical system is faster to simulate, and
- The closed-form dynamical equation can be used as a basis for model-based control design.

Typically, equivalents in the form of:

- Transfer function (McCabe et al, 2005)
- State-space (Perez and Fossen, 2009; Faedo et al, 2018<sup>6</sup>)
- Impulse response (de Prony, 1795)

are produced, using time domain (Prony's method) or freq. domain fitting.

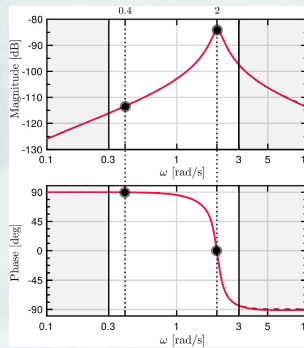
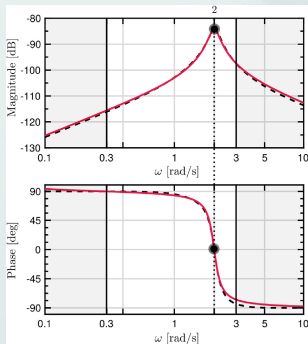
<sup>6</sup>Faedo, N., Pena-Sanchez and Ringwood, J.V. Finite-order hydrodynamic model determination for wave energy applications using moment matching, Ocean Engineering, Vol.163, pp 251-263, 2018.



# The FOAMM Toolbox



## FOAMM - Finite Order Approximation using Moment Matching



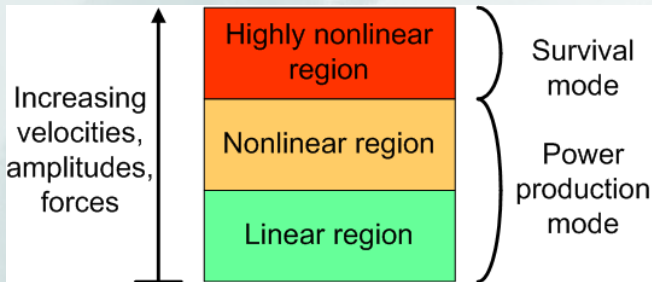
Force-to-velocity frequency response computed with NEMOH (dashed-black) and the reduced order model (solid-red), considering one/two interpolation points.



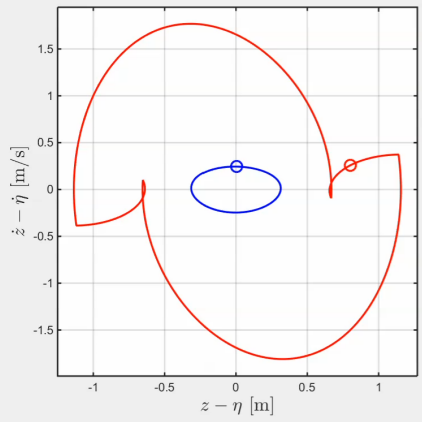
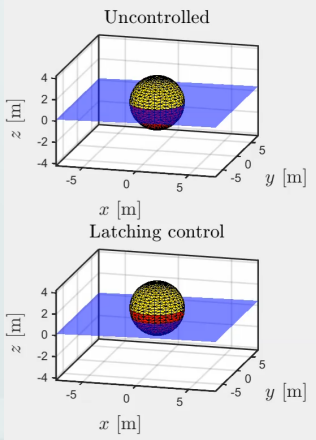
# WEC operating regions

Possible nonlinearities:

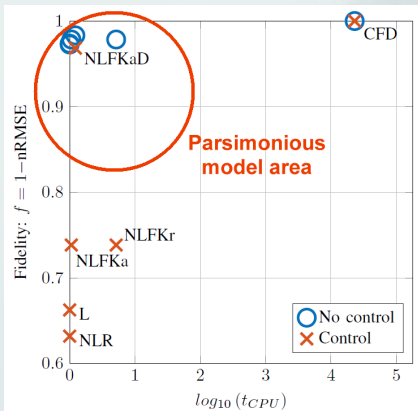
- Nonlinear fluid/structure interactions
- Nonlinear PTO system
- Nonlinear waves



# Linearisation ?



# Comparison of modelling methodologies<sup>7</sup>



**L:** A fully linear model, considering the mean wetted surface

**NLR:** A nonlinear static FK force model, using the instantaneous restoring force

**NLFKa:** A nonlinear static and dynamic FK forces model, using the algebraic solution of the pressure integral over the instantaneous wetted surface

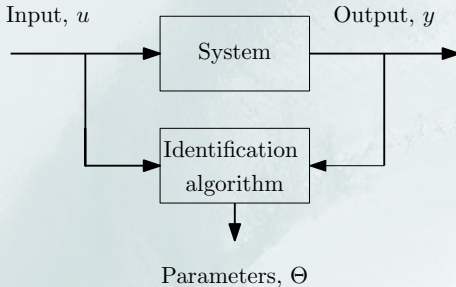
**NLFKr:** A nonlinear static and dynamic FK forces model, using a discretized geometry and a re-meshing routine to determine the instantaneous wetted surface

**NLFKaD:** An algebraic nonlinear static and dynamic FK forces model with a viscous drag term

**CFD:** A fully-nonlinear model, using a computational fluid dynamics software

<sup>7</sup>Giorgi, G. and Ringwood, J.V. Nonlinear hydrodynamic modelling for wave energy devices in the computation/fidelity continuum, Ocean Engineering, Vol.141, pp 164-175, 2017

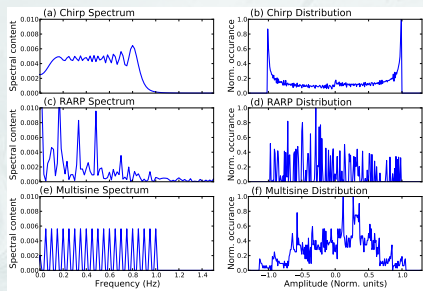
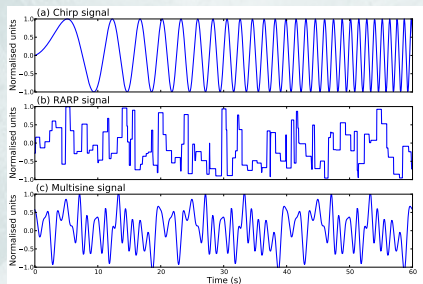
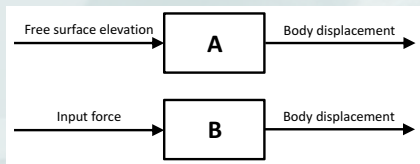
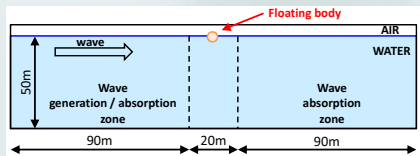
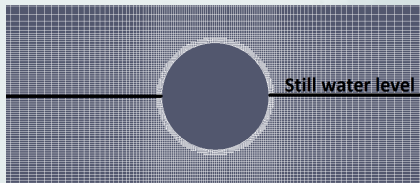
# Hydrodynamic models from data



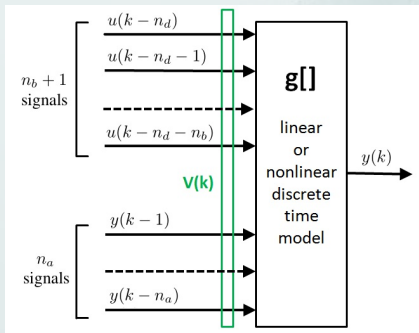
Need to consider:

- How the device output/motion is calculated/measured (NWT or tank/ocean)
- What type of excitation should/can be generated
- How the model will be parameterised, and
- How the model parameters will be identified

# Data from numerical wave tank



# Model parameterisation



Kolmogorov-Gabor (KGP) model:

polynomial

$$\begin{aligned}
 y(k) &= \sum_{i=1}^{n_a} a_{i1} y(k-i) + \sum_{i=0}^{n_b} b_{i1} u(k-n_d-i) \\
 &+ \dots \\
 &+ \sum_{i=1}^{n_a} a_{ip} y^p(k-i) + \sum_{i=0}^{n_b} b_{ip} u^p(k-n_d-i) \\
 &+ \sum_{i=1}^{n_a} \sum_{j=0}^{n_b} c_{ij} y(k-i) u(k-n_d-j) \\
 &+ \dots
 \end{aligned} \tag{7}$$

NWT and ID procedures documented in <sup>8 9</sup>

<sup>8</sup> Davidson, J., Giorgi, S. and Ringwood, J.V. Identification of wave tank models from numerical wave tank data Part 1: NWT identification tests. IEEE Trans. on Sustainable Energy, Vol.7, No.3, pp 1012-1019, 2016

<sup>9</sup> Giorgi, S., Davidson, J. and Ringwood, J.V. Identification of wave tank models from numerical wave tank data Part 2: Data-based model determin. IEEE Trans. on Sustainable Energy, Vol.7, No.3, pp 1020-1027, 2016

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# Control problem statement

The real control objective:

## Energy cost minimisation

Minimise the Levelised Cost of Energy (LCoE) over the WEC/project lifetime

$$LCoE = \frac{PV(CapEx) + PV(OpEx)}{PV(EP)} \quad , \quad PV(CF) = \sum_{y=y_0}^Y \frac{CF(y)}{(1 + R_d/100)^y} \quad (8)$$

usually distilled to:

## Energy maximisation

Maximise captured energy:

$$E_c = \int_0^T v(t)F_{PTO}(t)dt \quad (9)$$

subject to:

$$|z(t)| < z_{max} \quad (10)$$

$$|F_{PTO}(t)| < F_{max} \quad (11)$$

$$|v(t)| < v_{max} \quad (12)$$

# Control fundamentals

From (6),

$$\frac{V(\omega)}{F_{ex}(\omega) + F_{PTO}(\omega)} = \frac{1}{Z_i(\omega)}, \quad (13)$$

where  $Z_i(\omega)$  is the intrinsic WEC impedance:

$$Z_i(\omega) = B_r(\omega) + j\omega \left[ M + M_a(\omega) - \frac{k}{\omega^2} \right], \quad (14)$$

with  $M_a(\omega)$  the added mass ( $m_\infty = \lim_{\omega \rightarrow \infty} M_a(\omega)$ ) and  $B_r(\omega)$  the radiation damping.

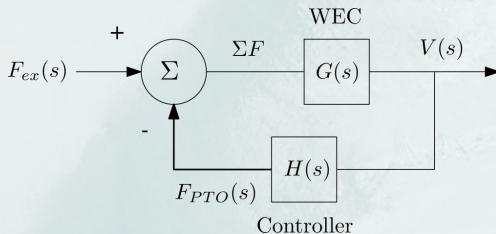
For maximum power transfer, we choose a controller 'impedance'  $Z_c(\omega)$ , ( $F_{PTO} = Z_c V$ ), so that

$$Z_c(\omega) = Z_i^*(\omega), \quad (15)$$

where  $z^*$  denotes the complex conjugate of  $z \in \mathbb{C}$ . Alternatively, an optimal velocity profile  $V_{opt}(\omega)$  to follow can be generated:

$$V_{opt}(\omega) = \frac{F_{ex}(\omega)}{2B_r(\omega)} \quad (16)$$

# ACC controller



Parameterise the control force as:

$$f_{PTO}(t) = M_c \ddot{x}(t) + B_c \dot{x}(t) + k_c x(t), \quad (17)$$

giving

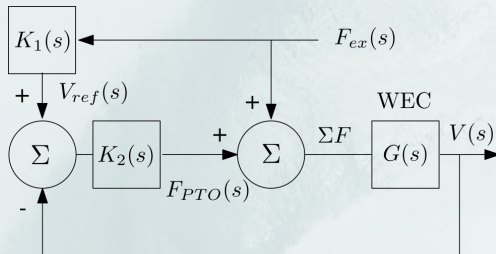
$$Z_c = F_{PTO}/V = B_c + j \left( \omega M_c - \frac{k_c}{\omega} \right), \quad H(s) \equiv Z_c(\omega) \quad (18)$$

$$G(s) = \frac{s}{(M + M_a^\omega)s^2 + B_r^\omega s + k}, \quad H(s) = \frac{-(M + M_a^\omega)s^2 + B_r^\omega s - k}{s} \quad (19)$$

and

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{2B_r^\omega}, \quad \rightarrow V(s) = T(s)F_{ex}(s) = \frac{F_{ex}(s)}{2B_r^\omega} \quad (20)$$

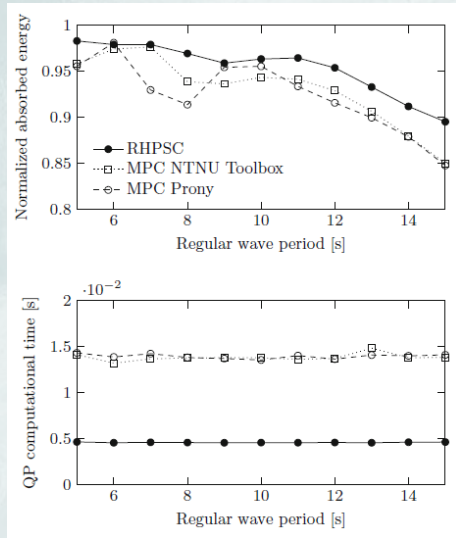
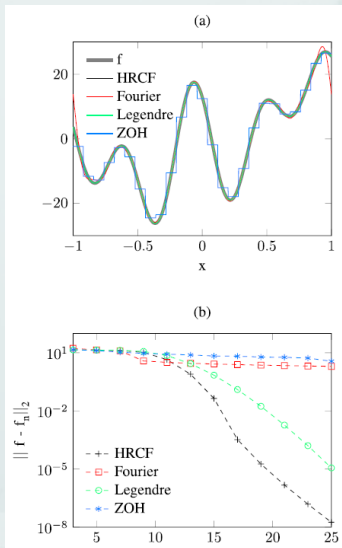
# AVT controller



- Effectively implements a version of  $V_{opt}(\omega) = \frac{F_{ex}(\omega)}{2B_r(\omega)}$  i.e. (16)
- $v_{ref}(t)$  is usually evaluated as the solution of a numerical optimisation problem<sup>10</sup> for panchromatic case
- Since  $K_1$  is, in general, anticausal, future knowledge of  $f_{ex}(t)$  is required
- Can include physical constraints in optimisation problem
- Can apply robust synthesis to velocity tracking loop

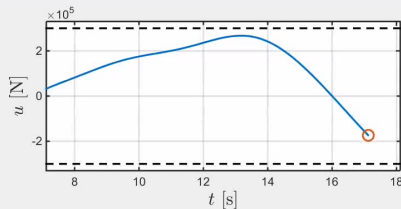
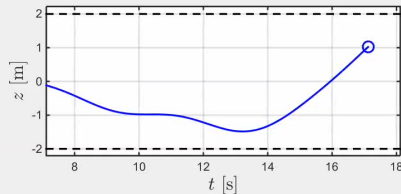
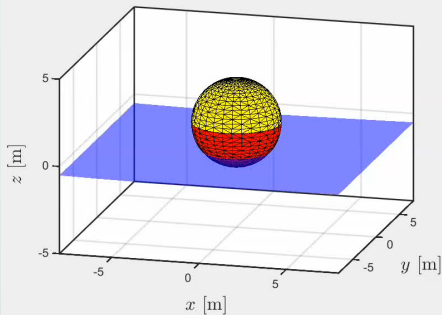
<sup>10</sup>Faedo, N., Olaya, S., and Ringwood, J.V. (2017). Optimal control, MPC and MPC-like algorithms for wave energy systems: An overview. IFAC Journal of Systems and Control, Vol.1, pp 37-56

# Signal parameterisation<sup>11</sup>



<sup>11</sup> Genest, R. and Ringwood, J.V. Receding horizon pseudospectral control for energy maximisation with application to wave energy devices, IEEE Trans. on Control Systems Technology, Vol.25, No.1, pp 29-38; 2017

# WEC control using moment matching<sup>12</sup>



<sup>12</sup>Faedo, N., Scarciotti, G., Astolfi, A. and Ringwood, J.V. Energy maximising control of wave energy devices using a moment-domain representation, Control Engineering Practice, Vol.81, pp 85-96, 2018

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# ACC controller calculations<sup>13</sup>



$$S_G^T(s) = \frac{dT(s)}{dG(s)} \frac{G(s)}{T(s)} = \frac{1}{1 + G(s)H(s)} = S_G^T(s) = \frac{(M + M_a^\omega)s^2 + B_r^\omega s + k}{2B_r^\omega s} \quad (21)$$

Also, since, in general,  $S_\alpha^T(s) = \frac{\alpha}{T(s)} \frac{dT(s)}{d\alpha} = S_G^T(s)S_\alpha^G(s)$ ,

$$S_{M^*}^G(s) = -\frac{M^*s^2}{M^*s^2 + B_r^\omega s + k} \quad (22) \quad S_{M^*}^T(s) = S_G^T(s)S_{M^*}^G(s) = -\frac{M^*s^2}{2B_r^\omega} \quad (25)$$

$$S_B^G(s) = -\frac{B_r^\omega s}{M^*s^2 + B_r^\omega s + k} \quad (23) \quad S_B^T(s) = S_G^T(s)S_B^G(s) = -\frac{1}{2} \quad (26)$$

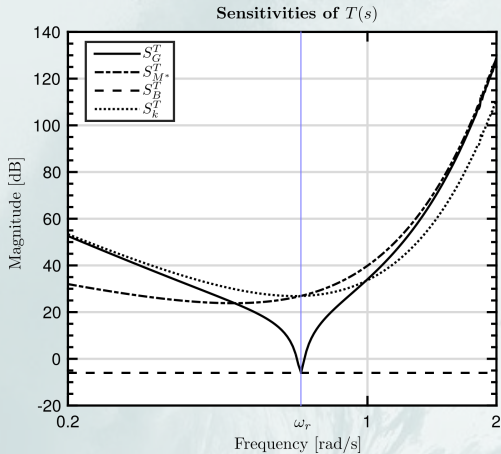
$$S_k^G(s) = -\frac{k}{M^*s^2 + B_r^\omega s + k} \quad (24) \quad S_k^T(s) = S_G^T(s)S_k^G(s) = -\frac{k}{2B_r^\omega s} \quad (27)$$

Note that  $M^* = M + M_a^\omega$

<sup>13</sup> Ringwood, J.V., Merigaud, A., Faedo, N. and Fusco, F. Wave energy control systems: Robustness issues, Proc. 11th IFAC Conference on Control Applications in Marine Systems, Robotics, and Vehicles (CAMS), Opatija, Croatia, Sept. 2018, pp 62-67



# ACC controller results



# Power sensitivity<sup>14</sup>



Define:

- $\rho_{\Re} := \frac{\Re\{\epsilon Z\}}{\Re\{Z_i\}}$  as the relative error in the radiation damping
- $\rho_{\Im} = \frac{\Im\{\epsilon Z\}}{\Im\{Z_i\}}$  represents relative errors in either inertial or stiffness terms

Then, can evaluate  $P_{act}/P^o$ , to different error types, where

- $P^o$  is power converted for the nominal system, and
- $P_{act}$  the actual power converted under perturbed conditions.

<sup>14</sup> Ringwood, J.V., Merigaud, A., Faedo, N. and Fusco, F. An analytical and numerical sensitivity and robustness analysis of wave energy control systems, *IEEE Trans. on Control Systems Technology*, in press (available online)

# Damping term errors



With modelling errors on damping terms only, i.e. errors in  $\Re\{Z_i\}$ , get:

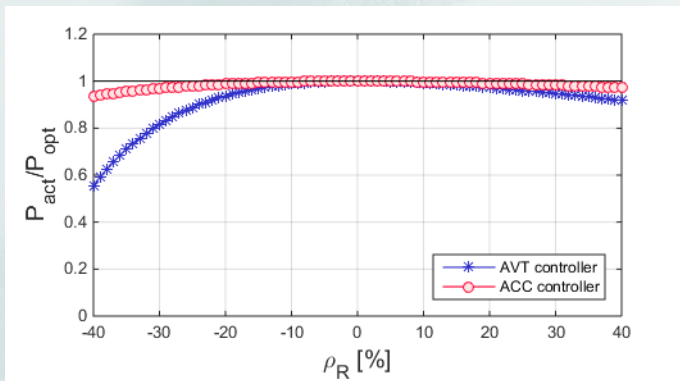
ACC

AVT

$$S_{\Re}(\rho_{\Re}) = \frac{1 + \rho_{\Re}}{1 + \rho_{\Re} + \frac{1}{4}\rho_{\Re}^2} \quad (28)$$

$$S_{\Re}(\rho_{\Re}) = \frac{1 + 2\rho_{\Re}}{(1 + \rho_{\Re})^2} \quad (29)$$

If  $\rho_{\Re} \ll 1$ ,  $S_{\Re}(\rho_{\Re}) \approx 1 - \frac{1}{4}\rho_{\Re}^2, \Rightarrow 10\% \rightarrow 0.25\%$       $S_{\Re}(\rho_{\Re}) \approx 1 - \rho_{\Re}^2, \Rightarrow 10\% \rightarrow 1\%$

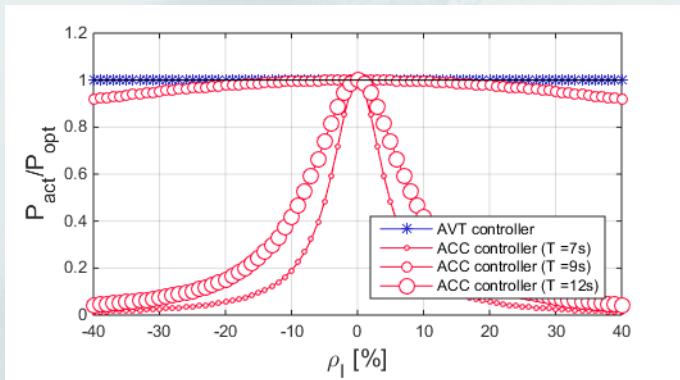


# Inertial or stiffness errors

With modelling errors on inertial or stiffness terms only, i.e. errors in  $\mathfrak{S}\{Z_i\}$ , get:

$$S_{\mathfrak{S}}(\rho_{\mathfrak{S}}) = \frac{\text{ACC}}{1 + \frac{1}{4} \frac{\mathfrak{S}\{Z_i\}^2}{\Re\{Z_i\}^2} \rho_{\mathfrak{S}}^2} \quad (30)$$

$$S_{\Re}(\rho_{\Re}) = 1 \quad \text{AVT} \quad (31)$$



Note that  $T_{res} \approx 9s$

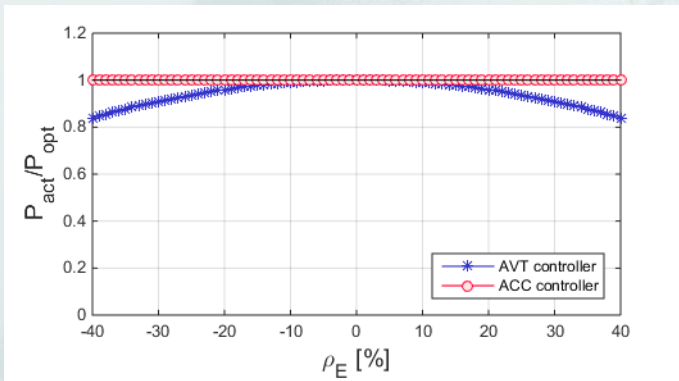
# Excitation force errors

ACC

$$S_E(\rho_E) = 1 \quad (32)$$

AVT

$$S_E(\rho_E) = 1 - |\rho_E|^2 \quad (33)$$



Note that  $\epsilon_E$  and  $F_{ex}$  are assumed to have the same phase over  $[0; \pi]$   
 i.e.  $\rho_E$  takes positive and negative real values

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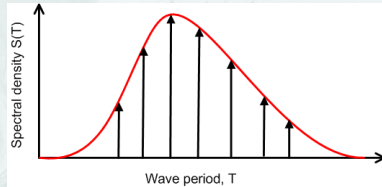
# Estimation of $F_{ex}$

$$\begin{bmatrix} \dot{z} \\ \dot{v} \\ \dot{\hat{F}}_e \\ \ddot{\hat{F}}_e \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_h}{m+A_\infty} & \frac{bh}{m+A_\infty} & \frac{1}{m+A_\infty} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\Omega^2 & 0 \end{bmatrix} \begin{bmatrix} z \\ v \\ \hat{F}_e \\ \dot{\hat{F}}_e \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} w \quad (34)$$

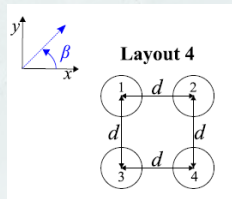
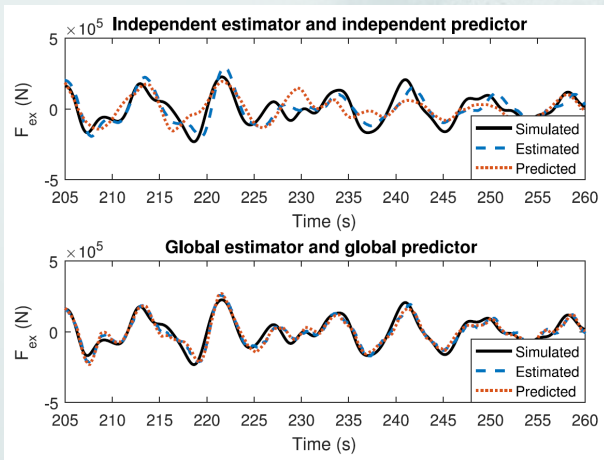
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ v \\ \hat{F}_e \\ \dot{\hat{F}}_e \end{bmatrix} + v \quad (35)$$

where

$$\Omega = \begin{bmatrix} \omega_1 & 0 & \dots & 0 \\ 0 & \omega_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_N \end{bmatrix} \quad (36)$$



# Estimation and forecasting in arrays<sup>15</sup>



7 frequencies  
in estimator  
AR model for  
forecasting  
(4 s)

<sup>15</sup>Pena, Y. Garcia-Abril, M., Paparella, F. and Ringwood, J.V. Estimation and forecasting of excitation force for arrays of wave energy devices, IEEE Trans. on Sustainable Energy, Vol.9, No.4, pp 1672-1680, Oct. 2018



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- Some rather spectacular sensitivity functions - nonlinear models required
- WEC arrays present both challenge and opportunity (measurement array)



# More info, and a few plugs!



**Ringwood, J.V., Bacelli, G. and Fusco, F.** Energy-maximising control of wave energy converters: The development of control system technology to optimise their operation, *IEEE Control Systems Magazine*, Vol.34, No.5, pp 30-55, Oct. 2014.



**Korde, U.A. and Ringwood, J.V.** *Hydrodynamic Control of Wave Energy Devices*, Cambridge University Press, 2016.

See also:

<http://www.eeng.nuim.ie/coer/publications/>



# Thank You !