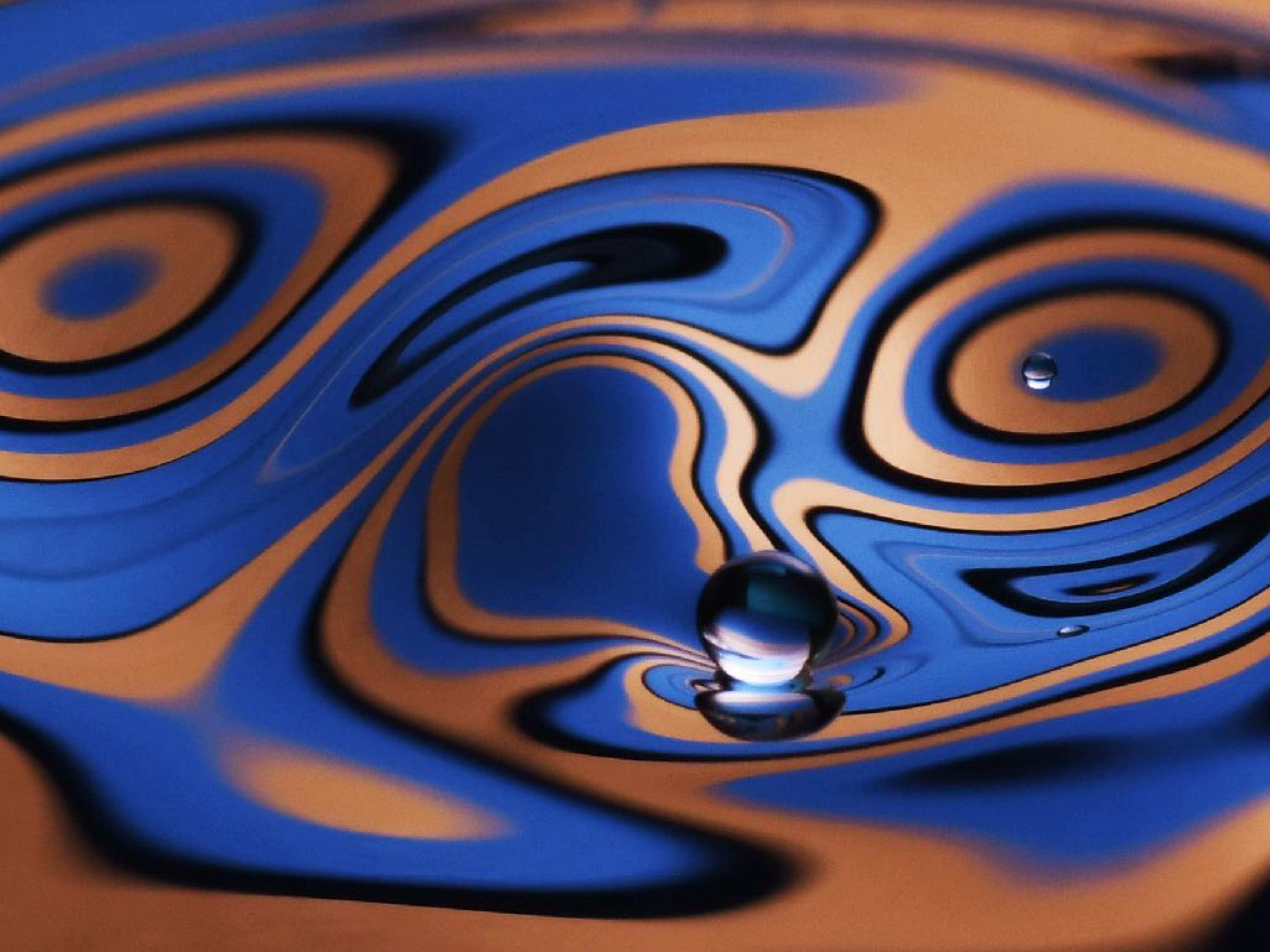


Bouncing and floating on a free surface: The kinematic match

Carlos Galeano-Rios

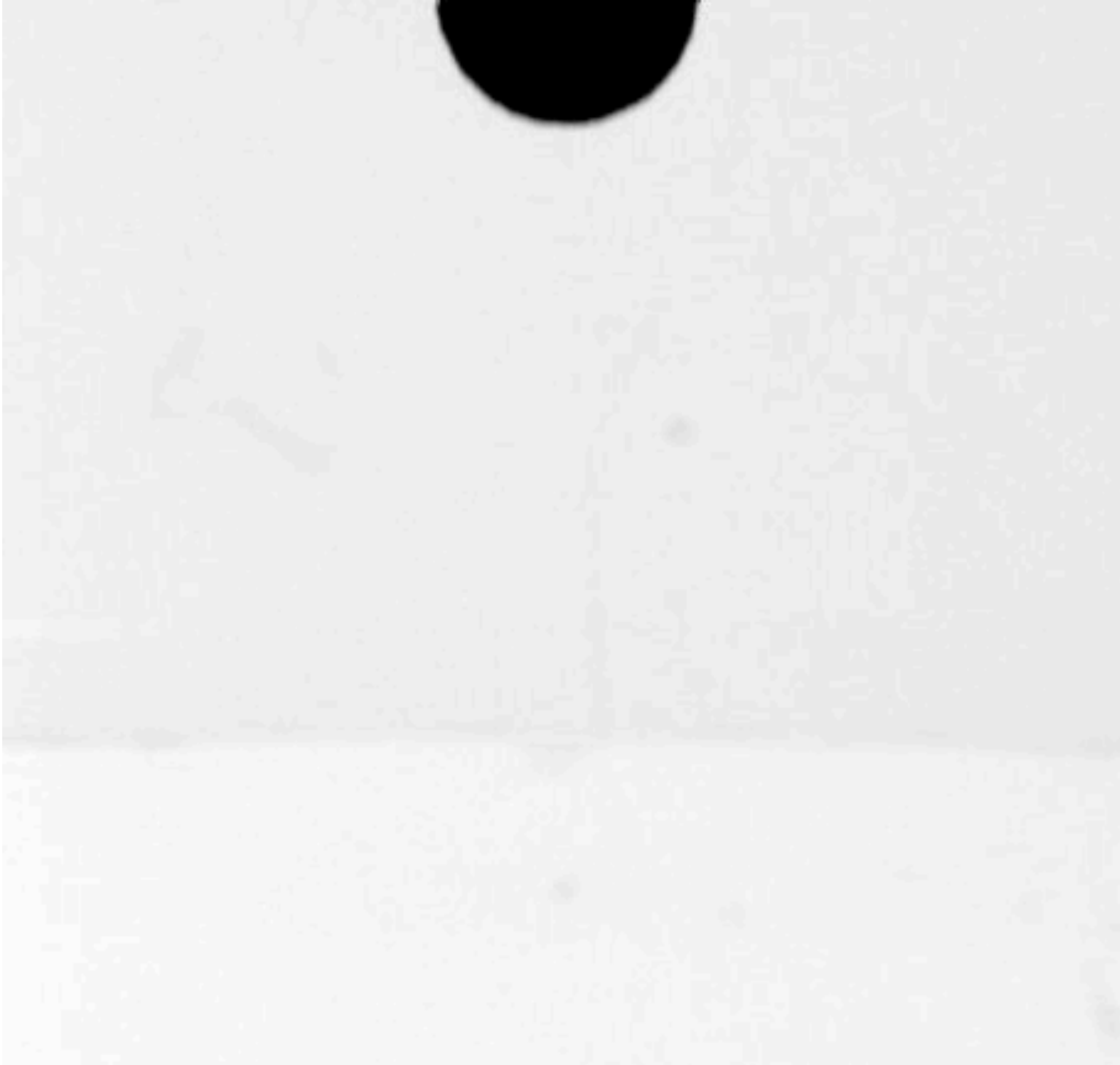
HyWEC 2

June 20th, 2019

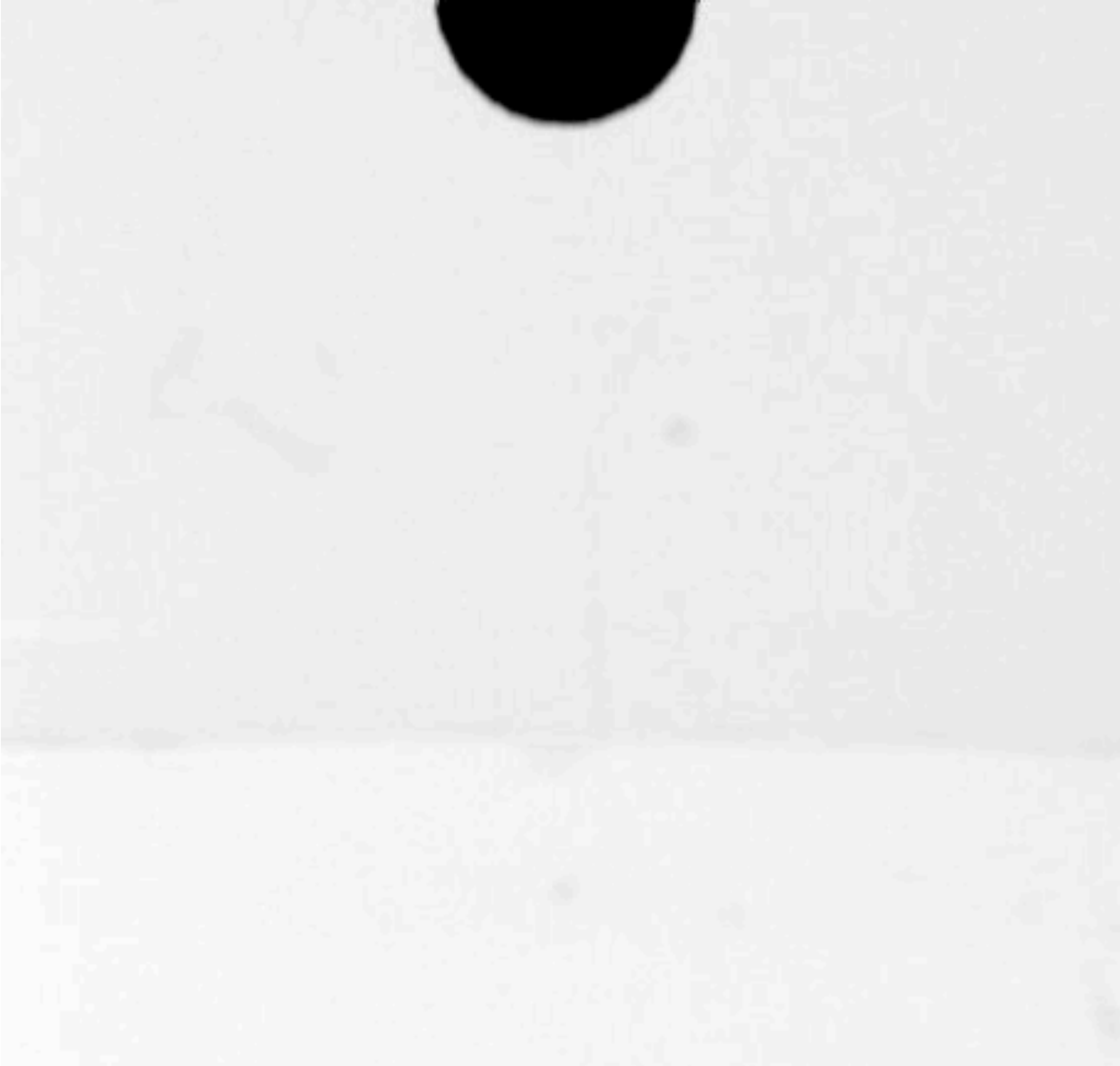
Collaborators: Paul Milewski (University of Bath)
Jean-Marc Vanden-Broeck (UCL)

UNIVERSITY OF BATH

EPSRC
Engineering and Physical Sciences Research Council



Experiments by Daniel Harris, Brown University



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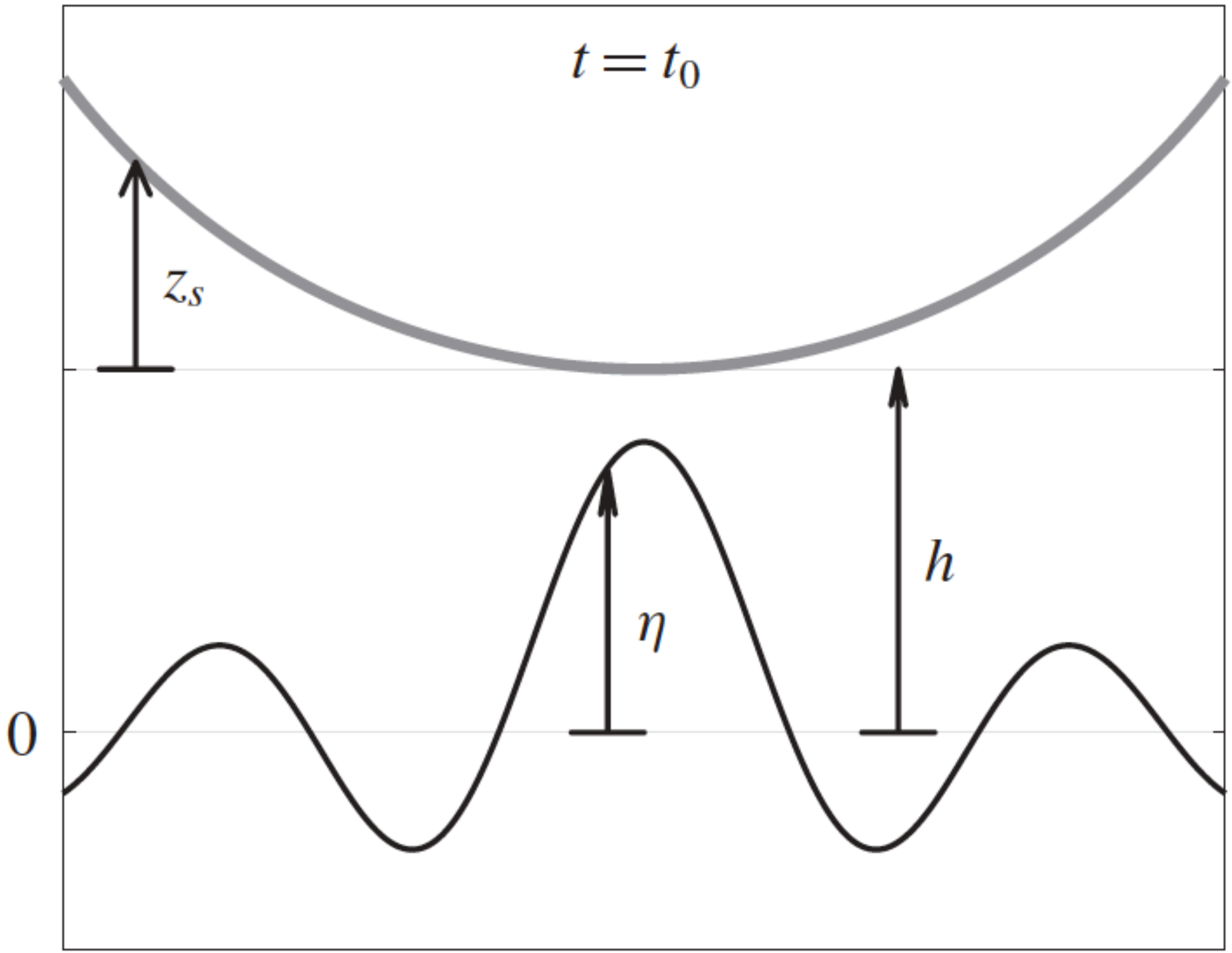
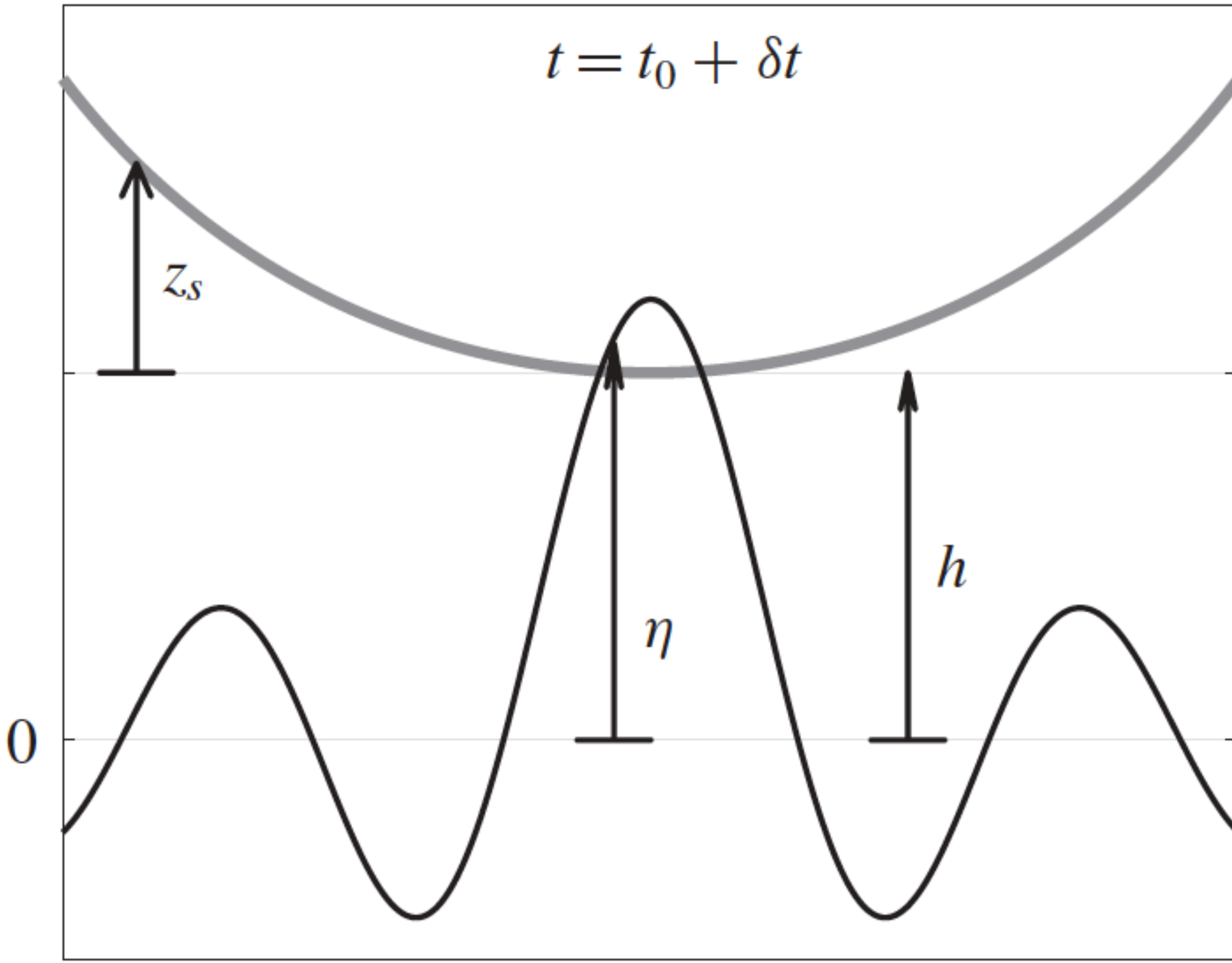
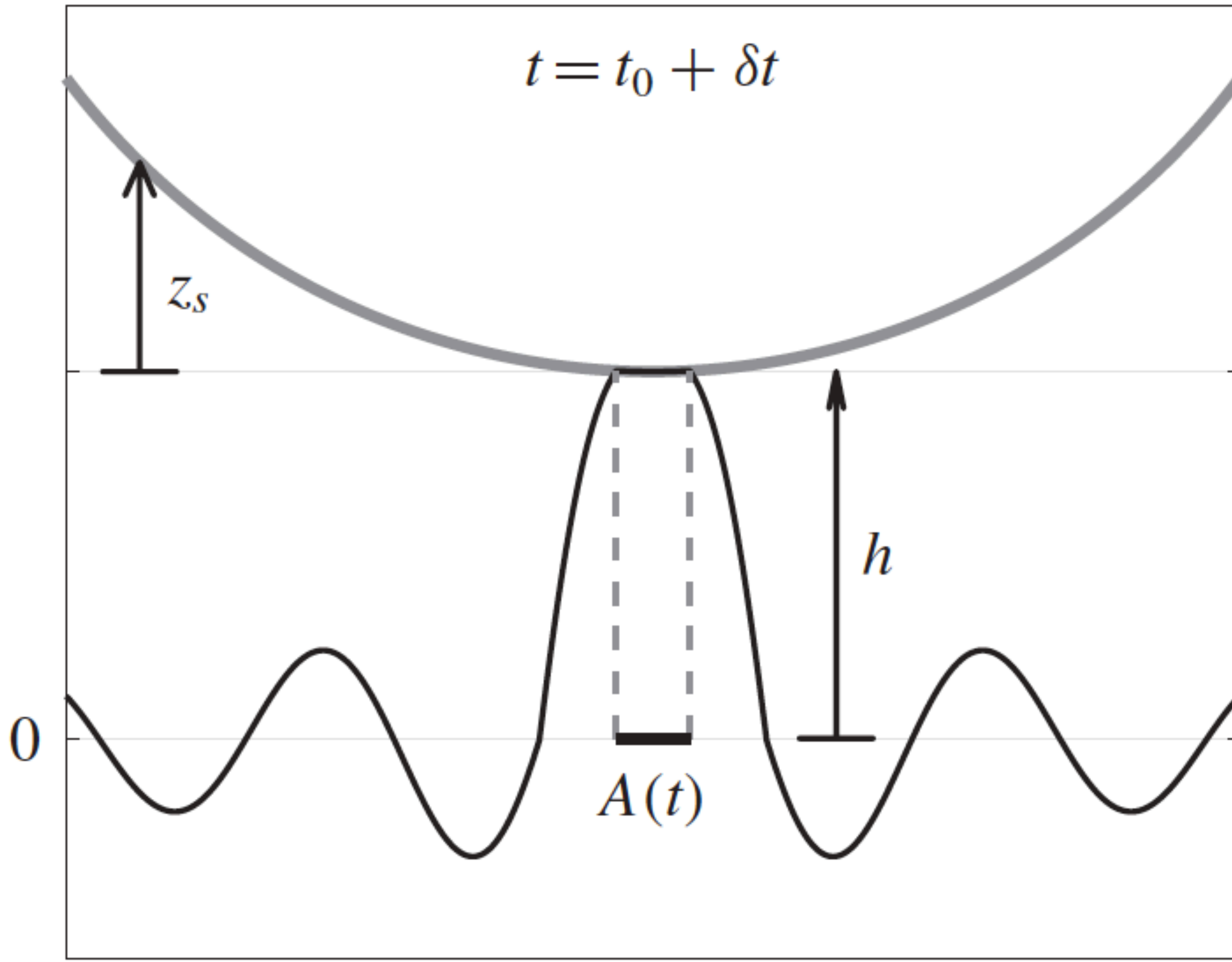
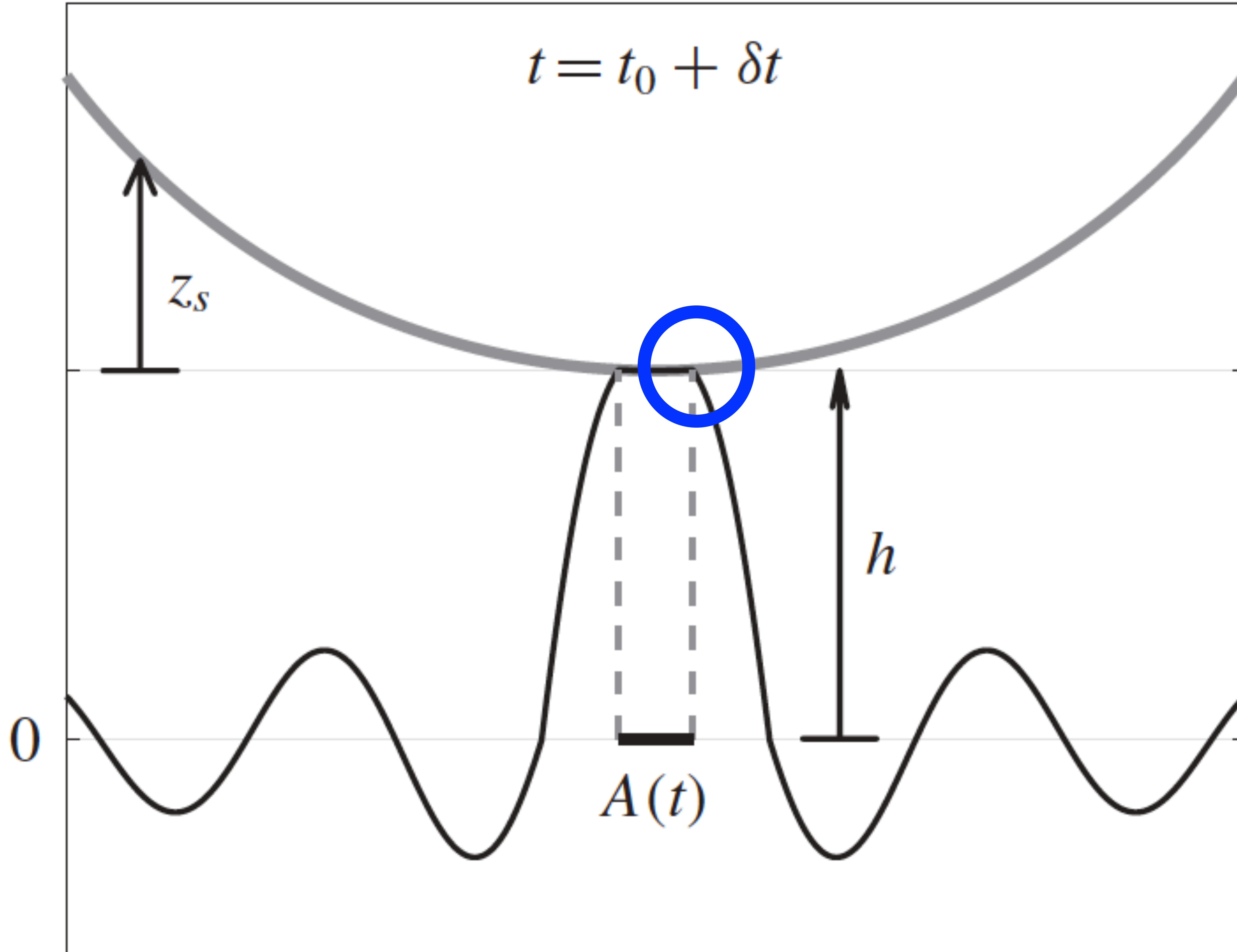


$t = t_0$
z_s
η
h
0


$t = t_0 + \delta t$
z_s
h
η
0


$t = t_0 + \delta t$
z_s
h
0
$A(t)$



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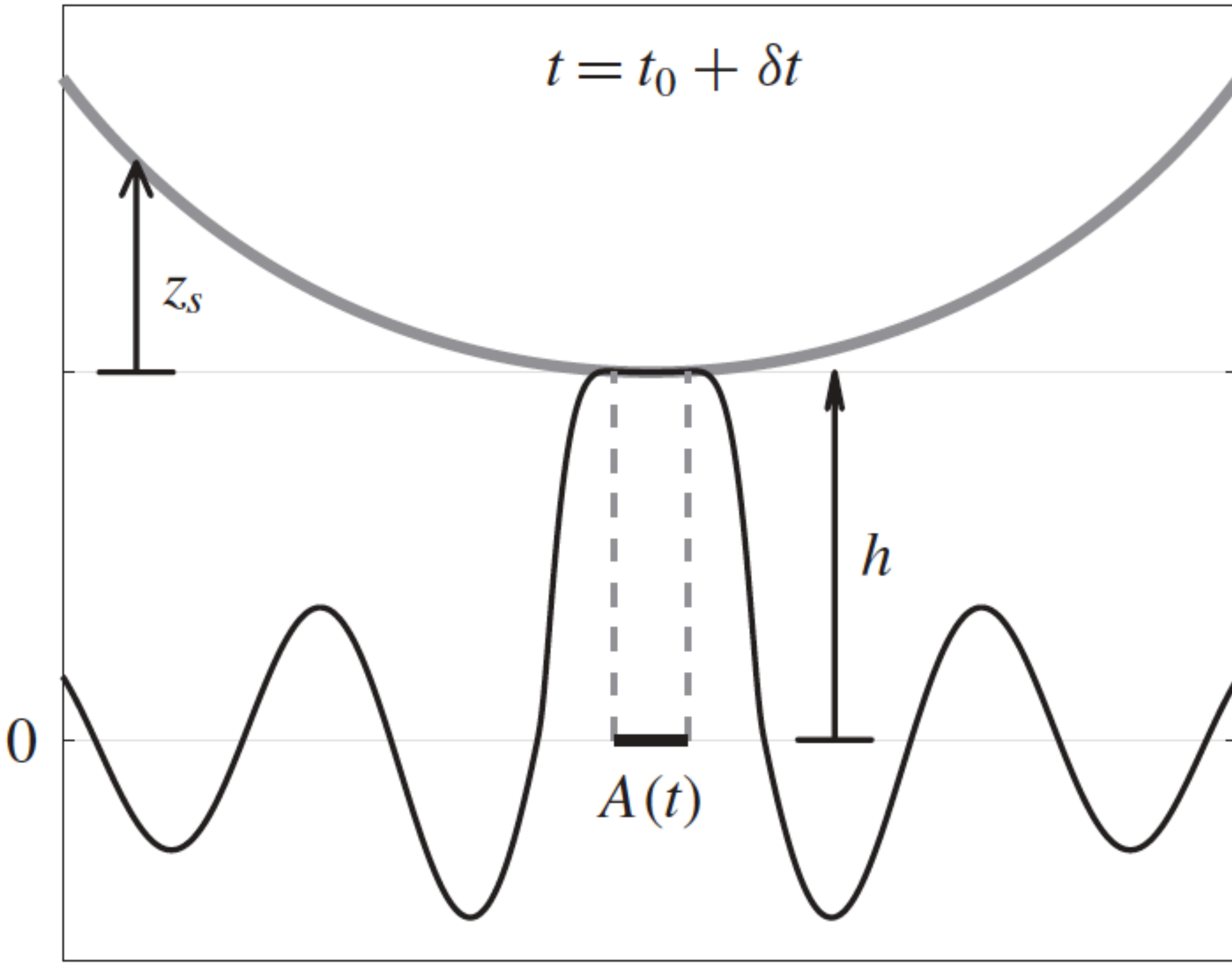
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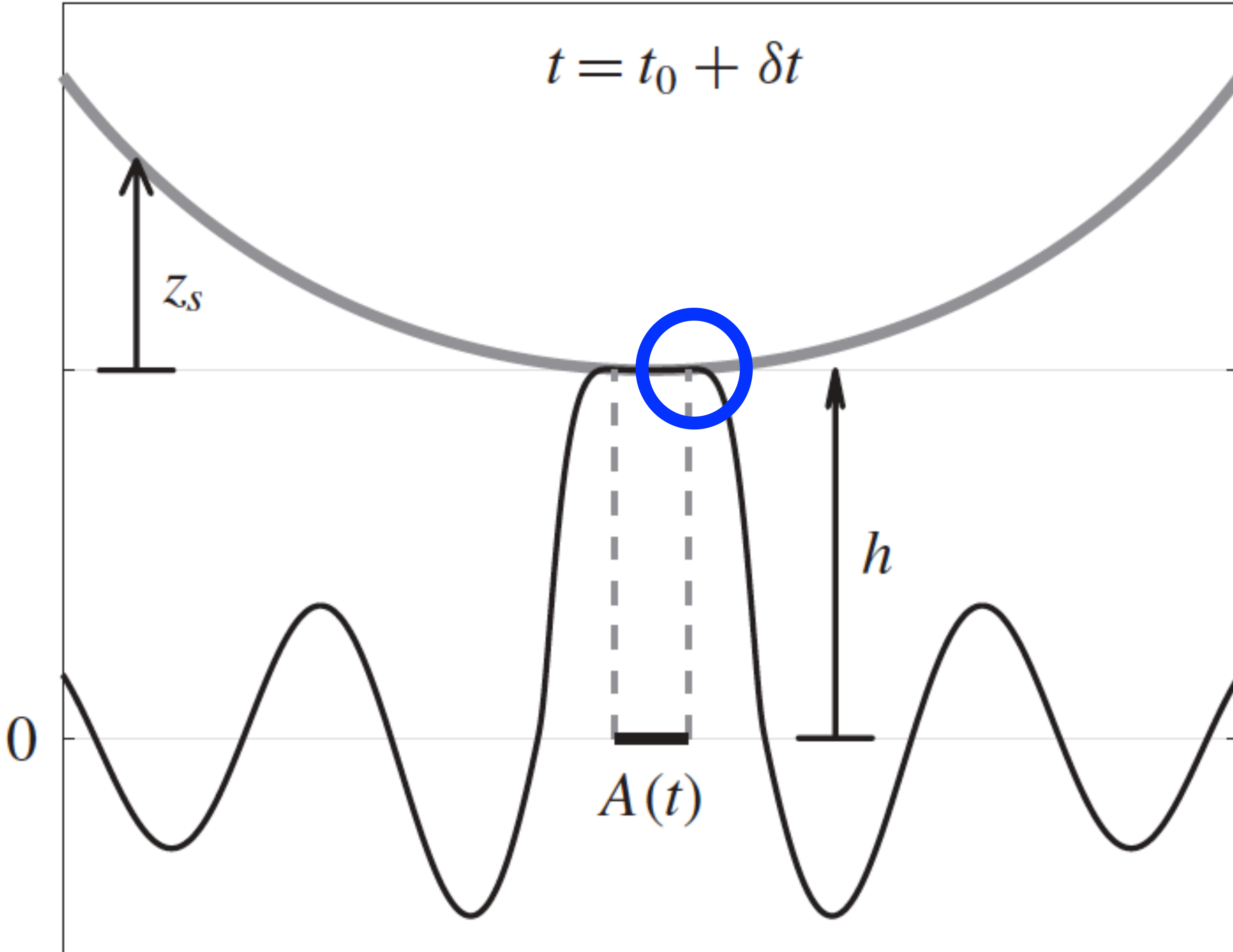

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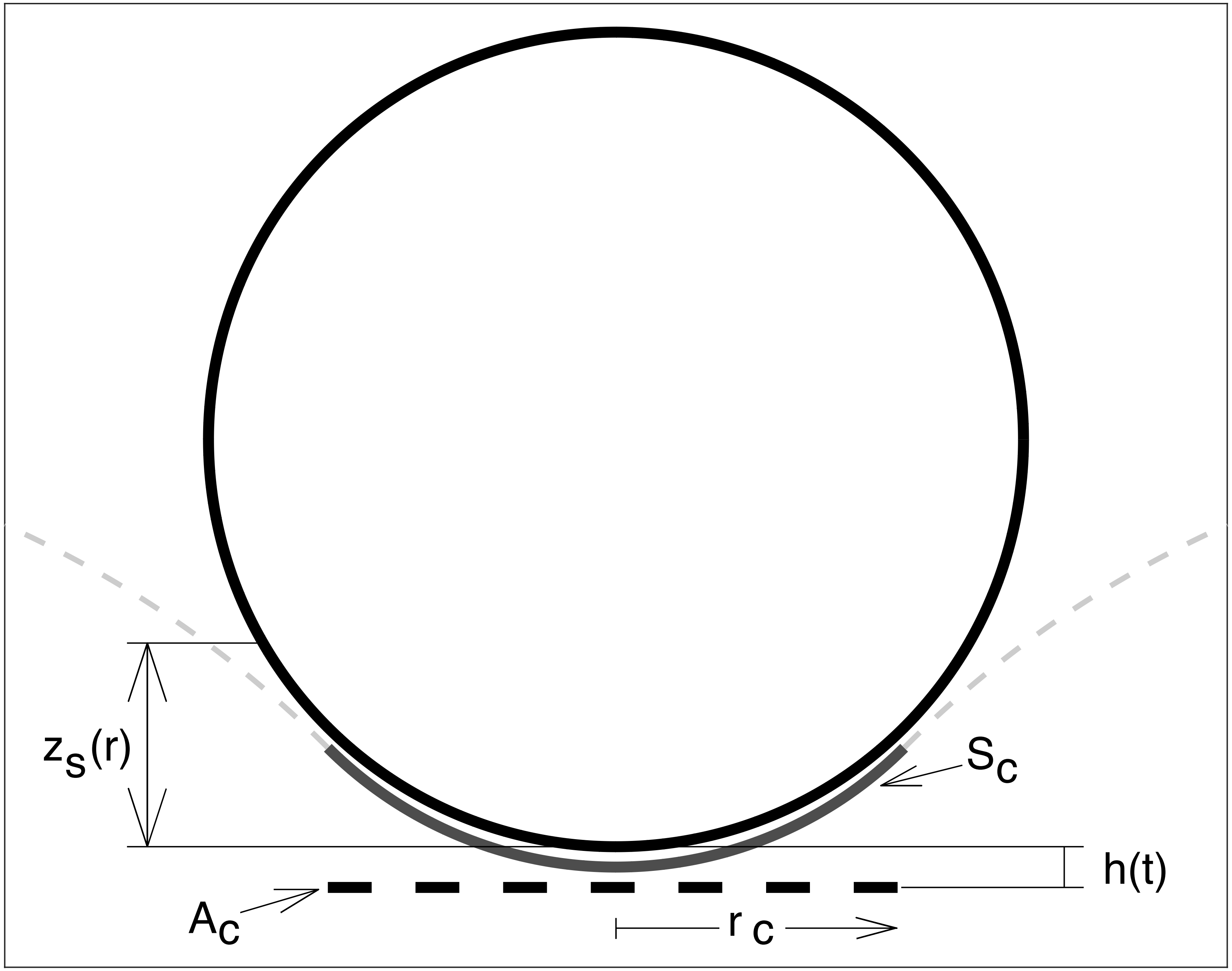
Falling Sphere = Moving Ceiling



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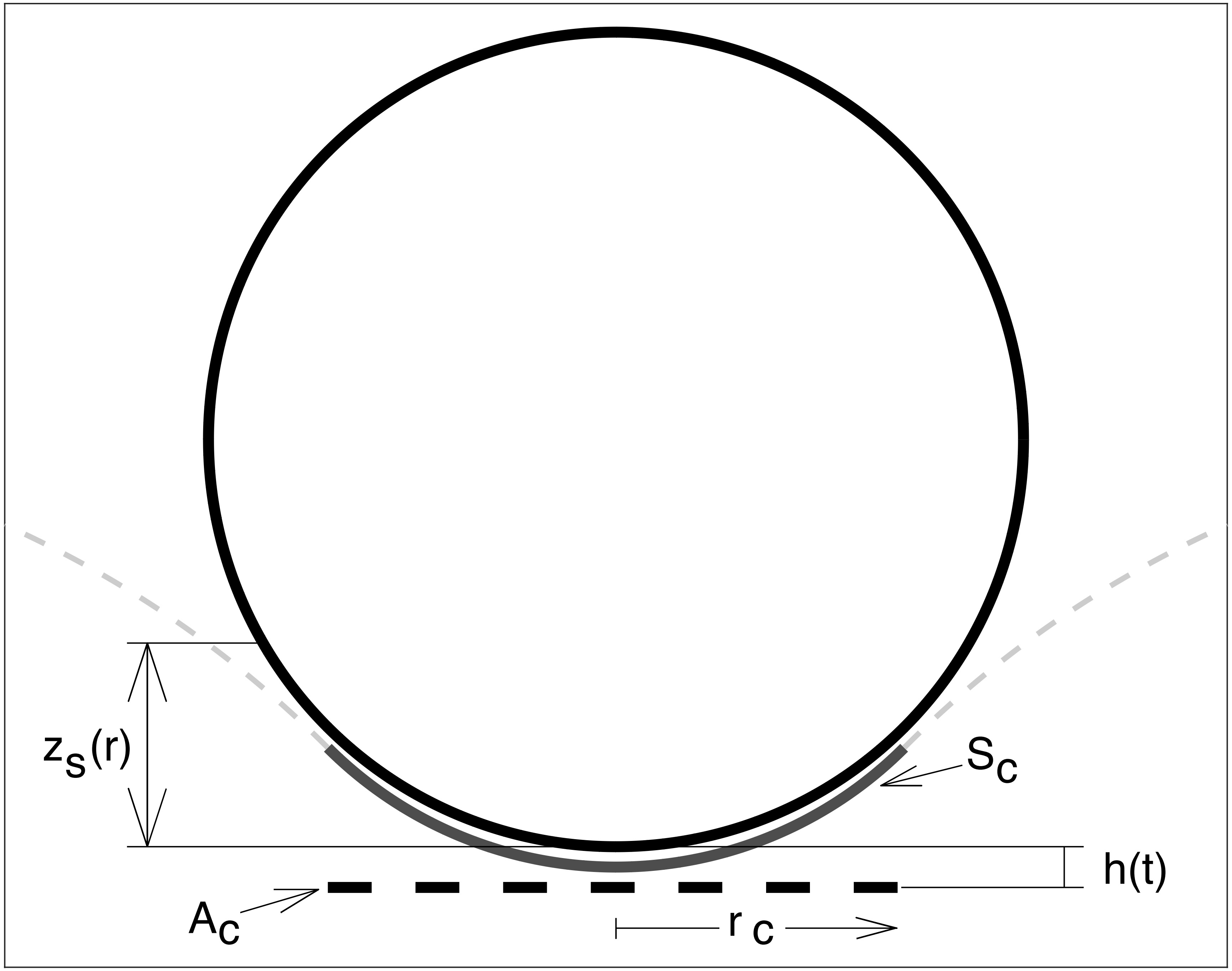
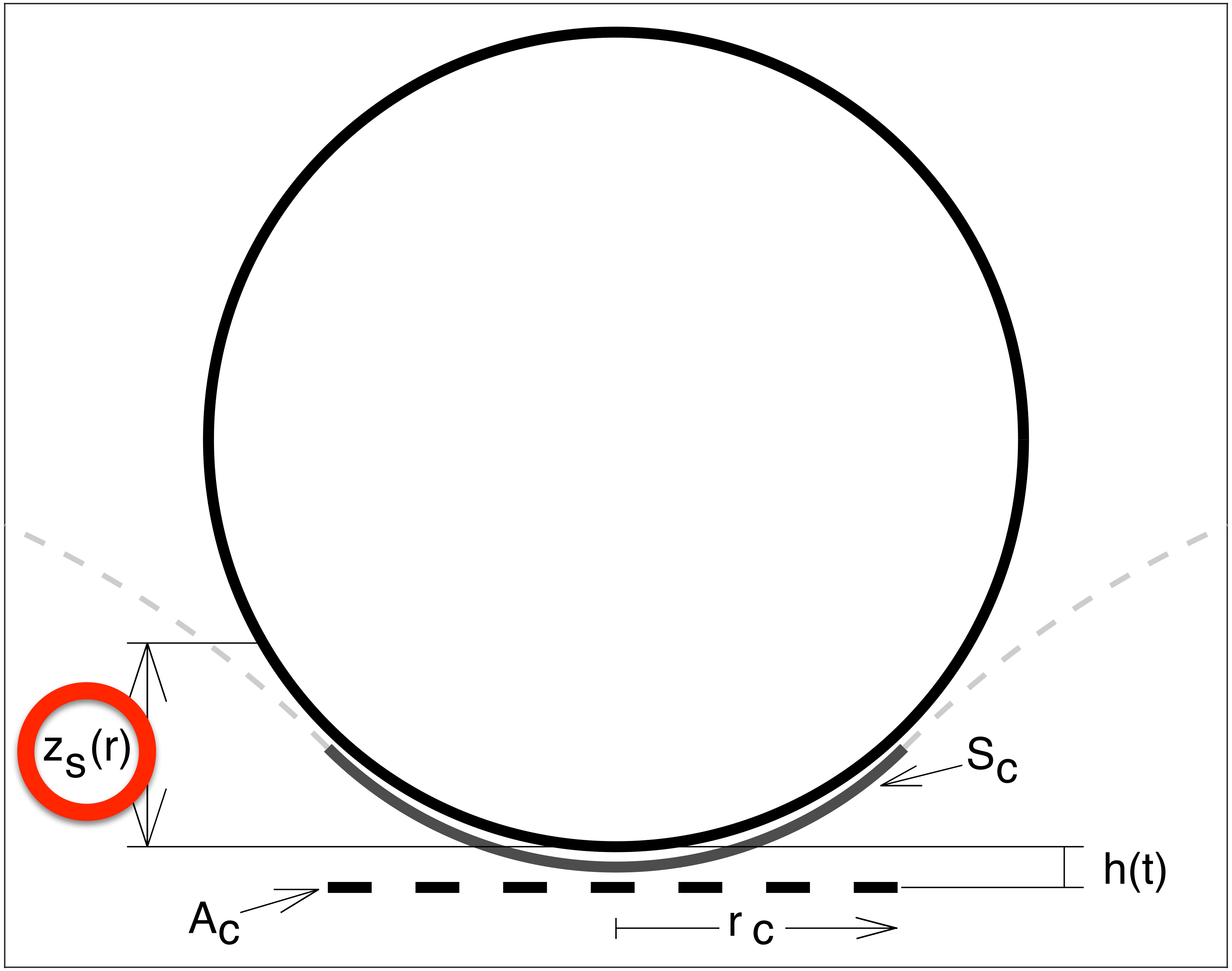


Perfectly
Hydrophobic
Rigid
Sphere

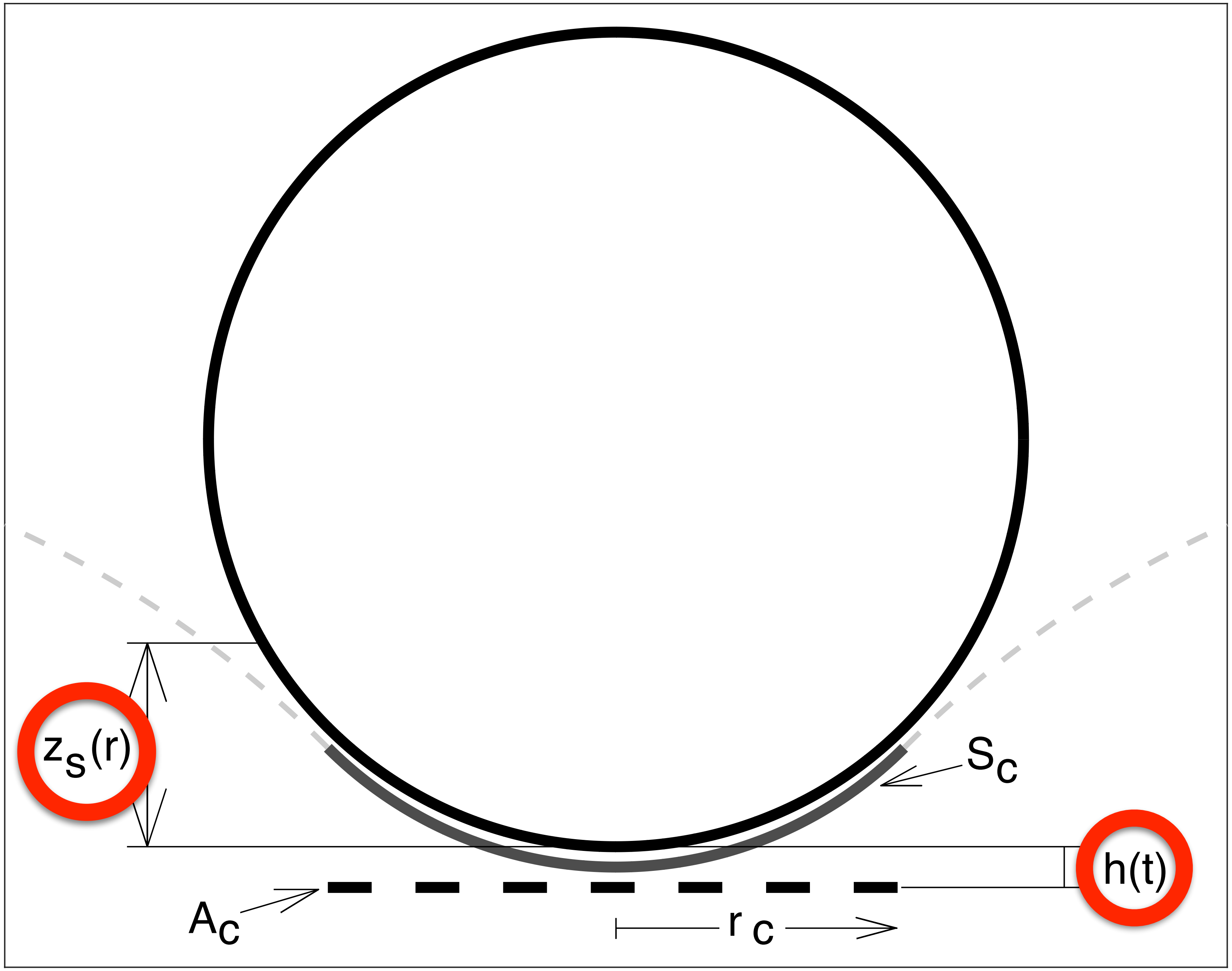
Falling Sphere = Moving Ceiling



in the contact area

Perfectly Hydrophobic Rigid Sphere

Falling Sphere = Moving Ceiling



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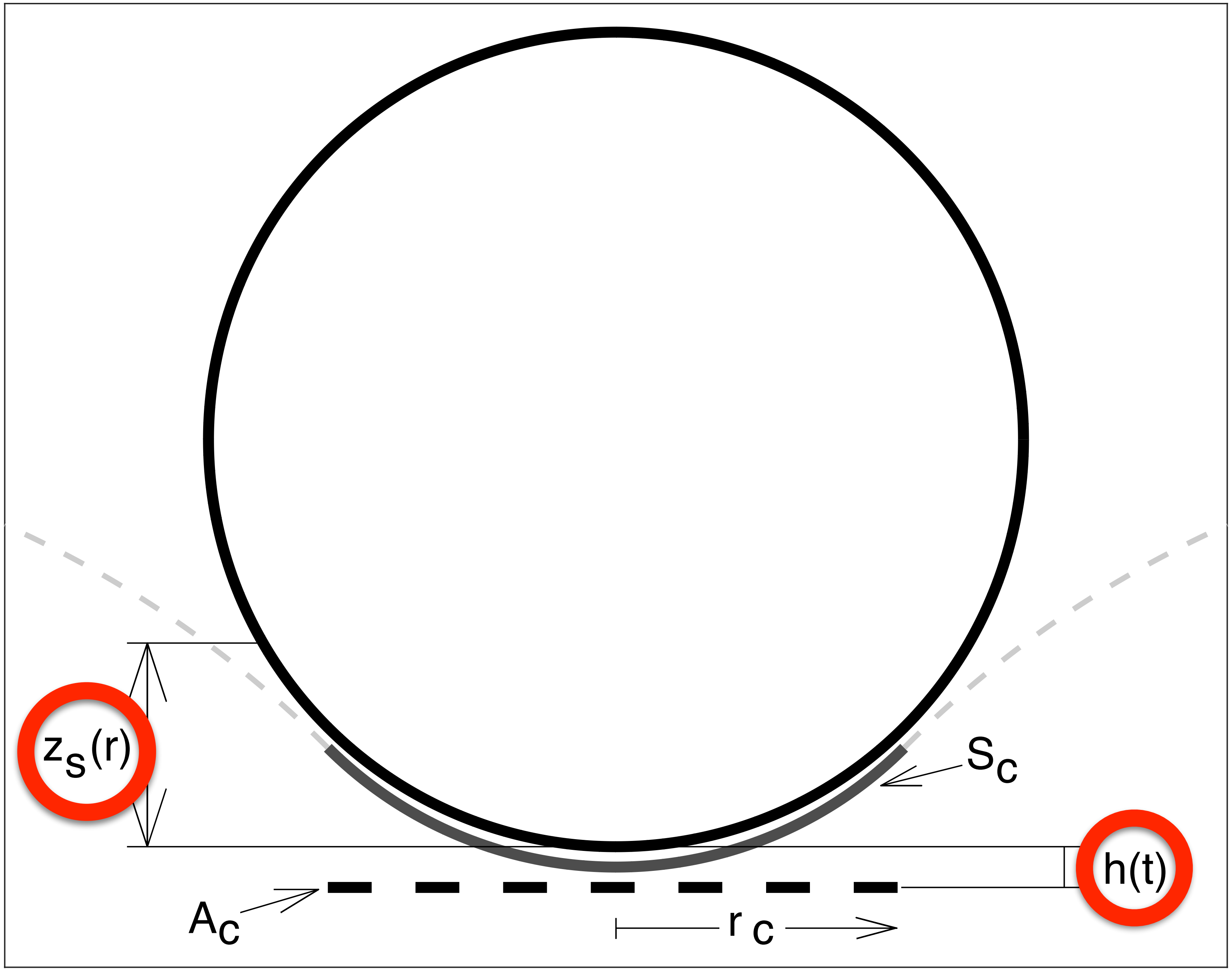

Perfectly Hydrophobic Rigid Sphere

in the contact area → $h(t) + z(x, y) = \eta(x, y, t)$

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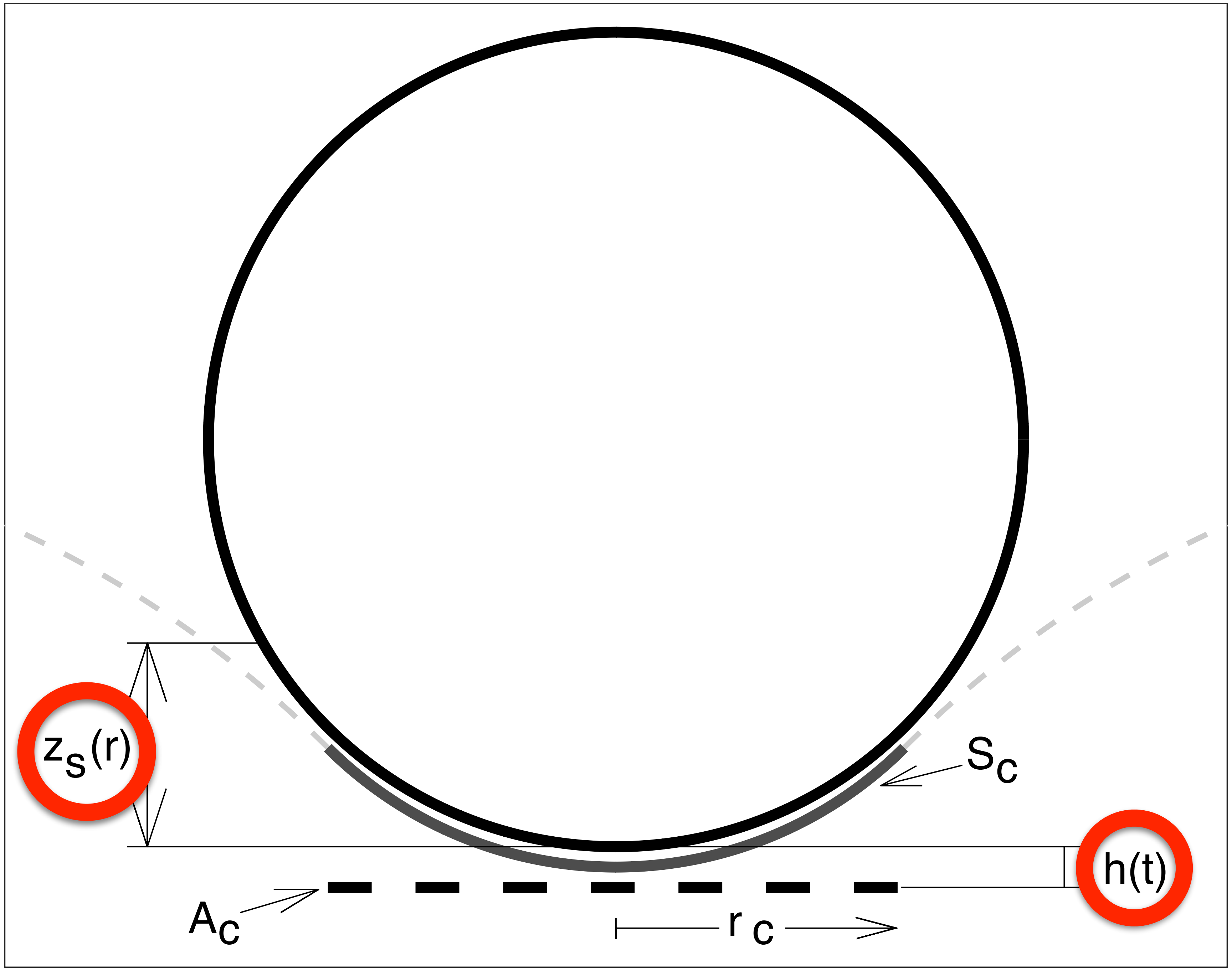

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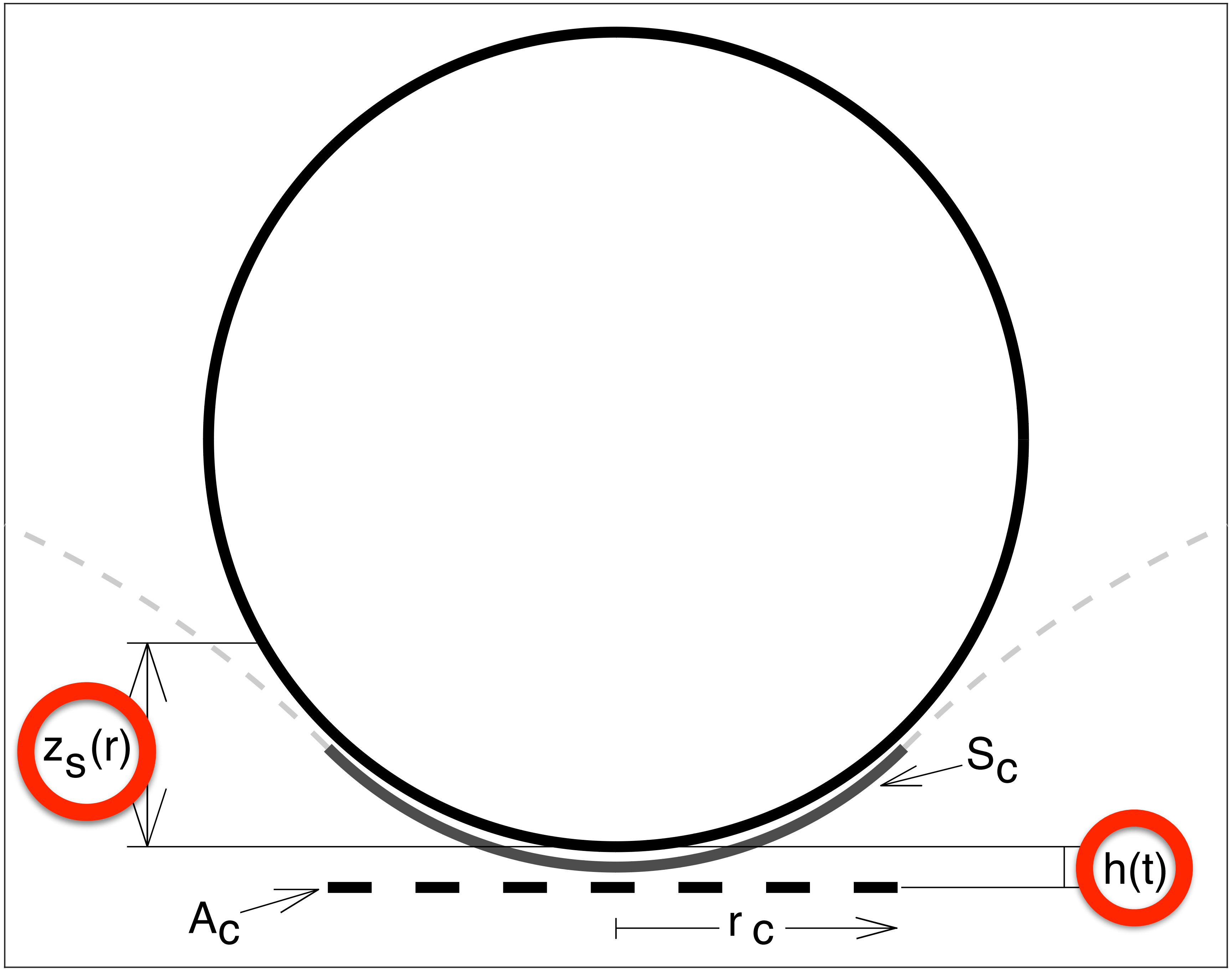

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0 fitting parameters

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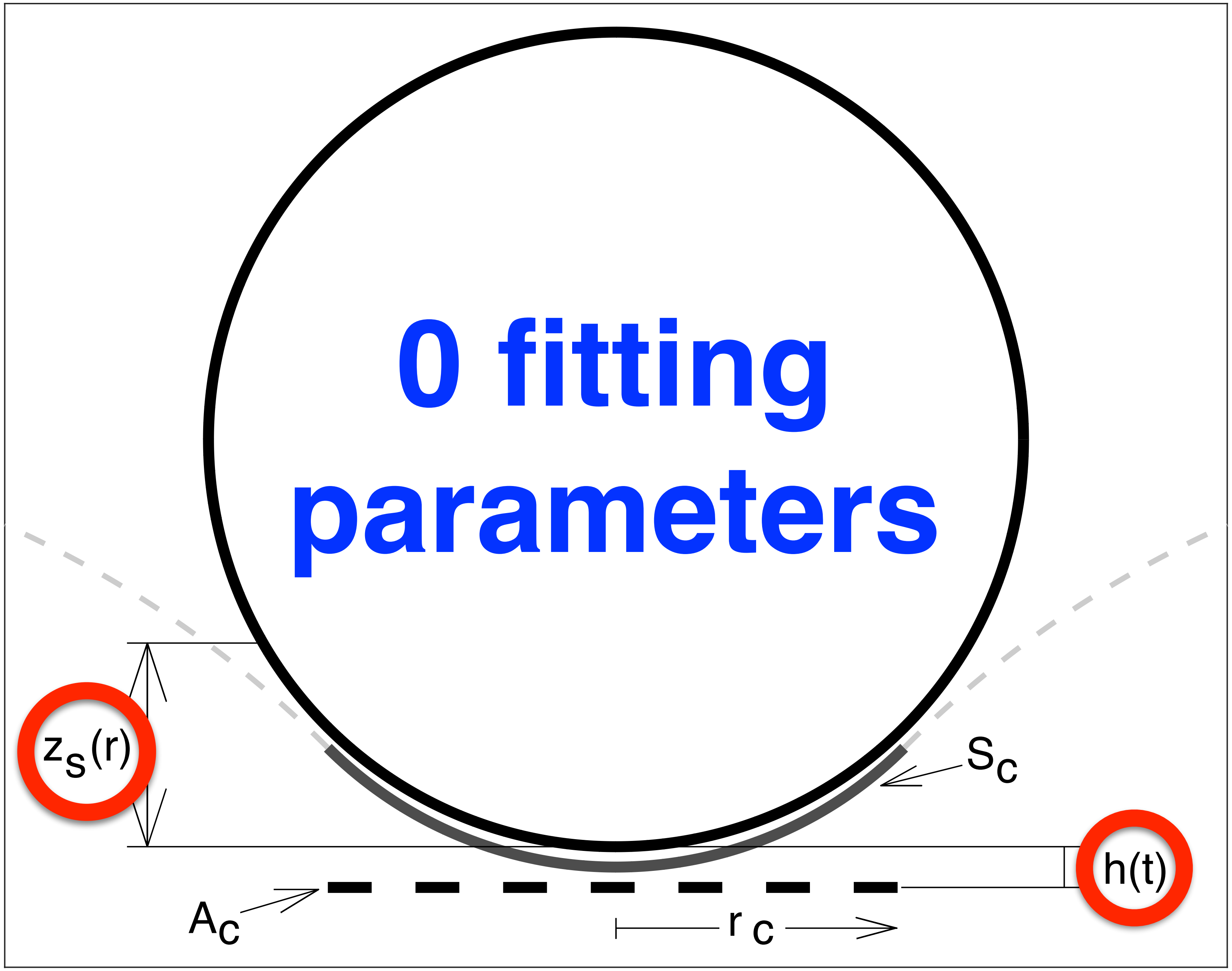

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A Non-local Formulation in Physical Space

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$$\Delta\phi = 0, \qquad z \leqslant 0,$$

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$$
\begin{aligned}
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\end{aligned}
$$

$$
\begin{aligned}
\phi &= \text{Velocity Potential} \\
\eta &= \text{Free Surface Elevation} \\
p_s &= \text{Pressure on Free Surface} \\
\Delta_H &= \partial_{xx} + \partial_{yy}
\end{aligned}
$$

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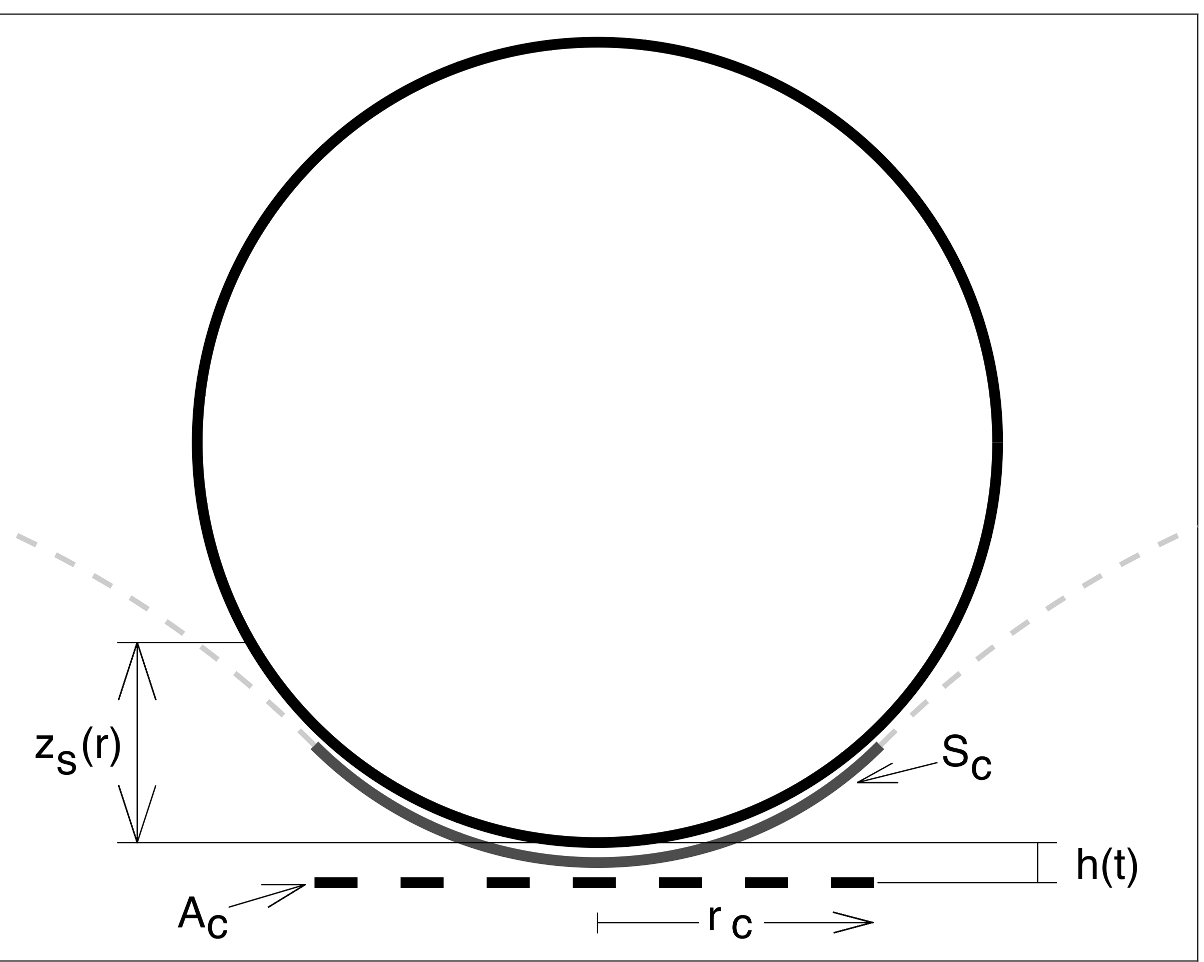

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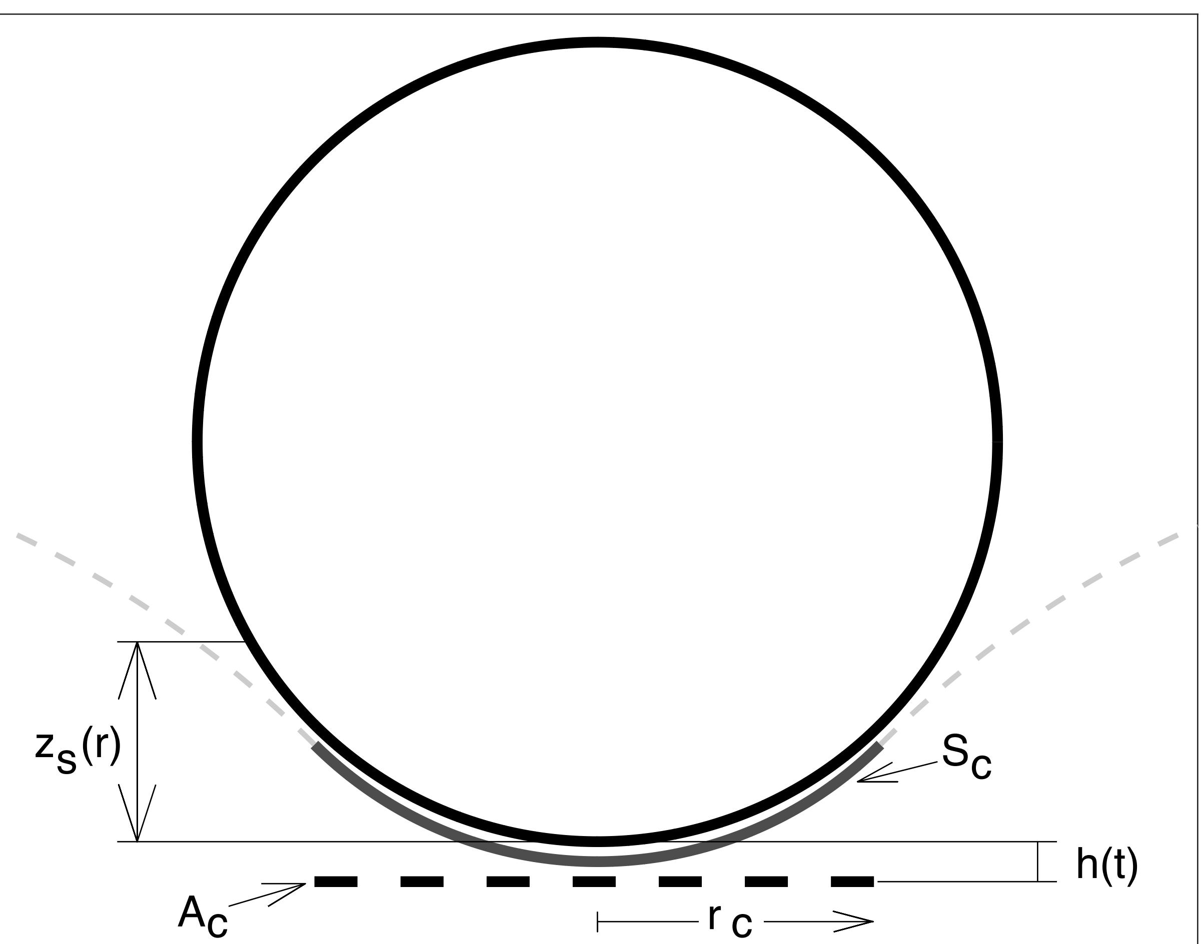

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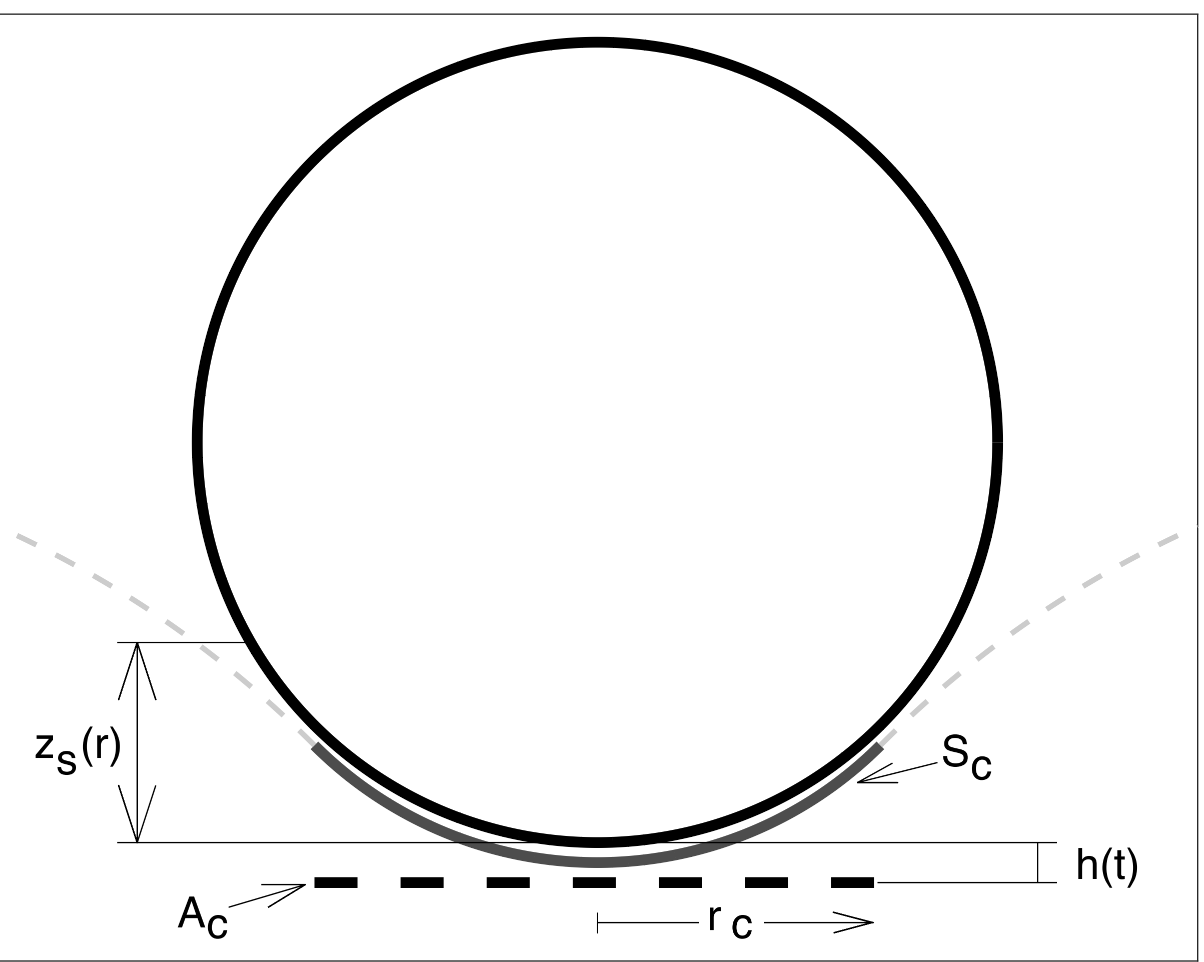

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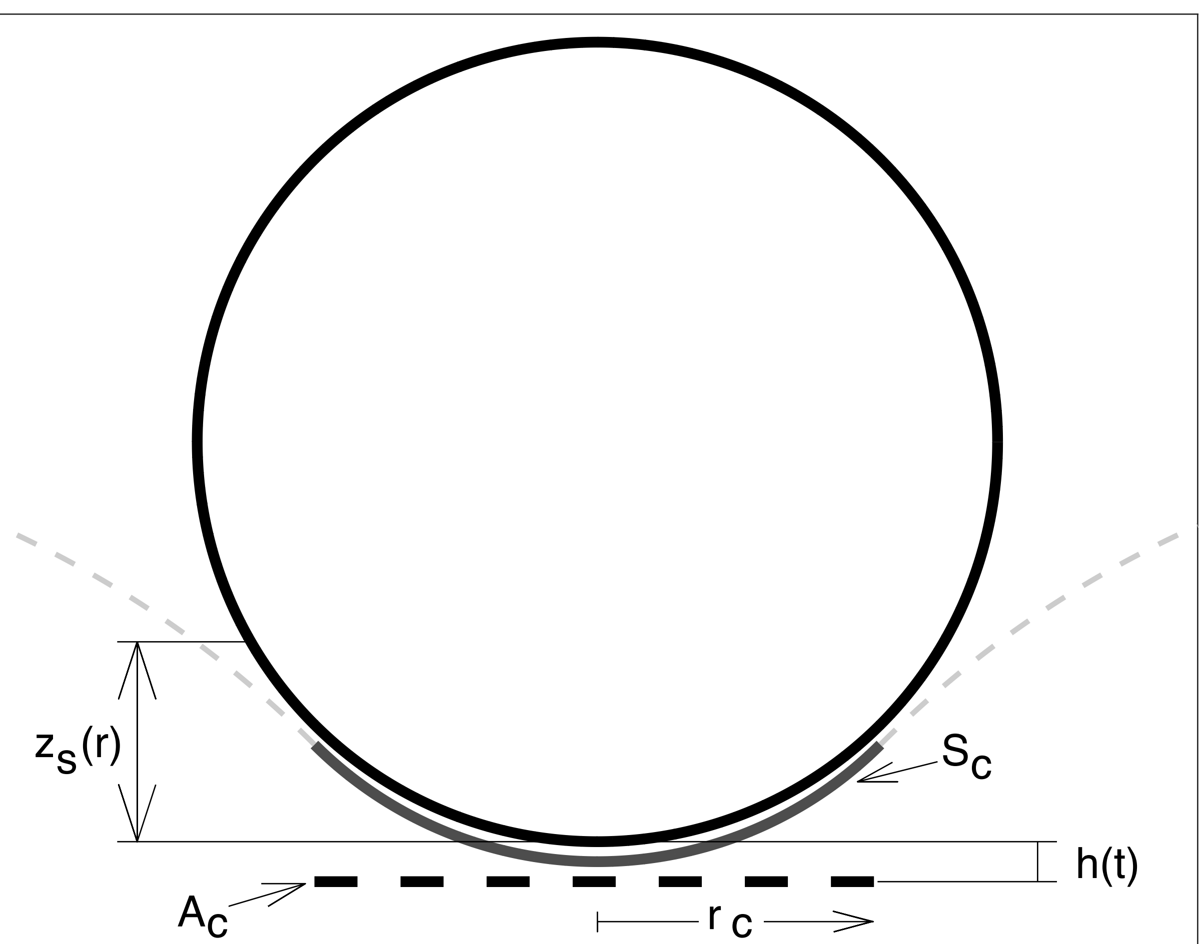

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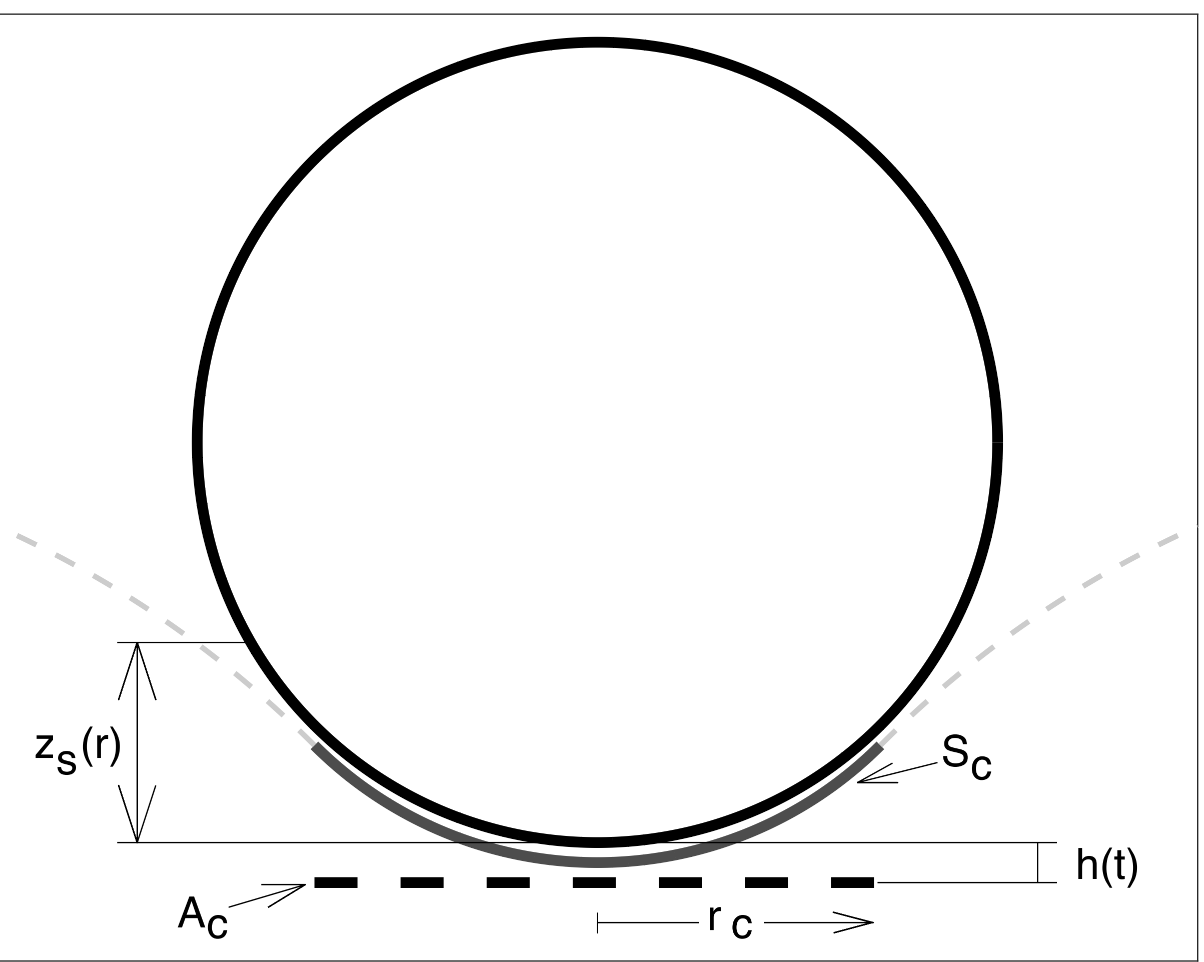

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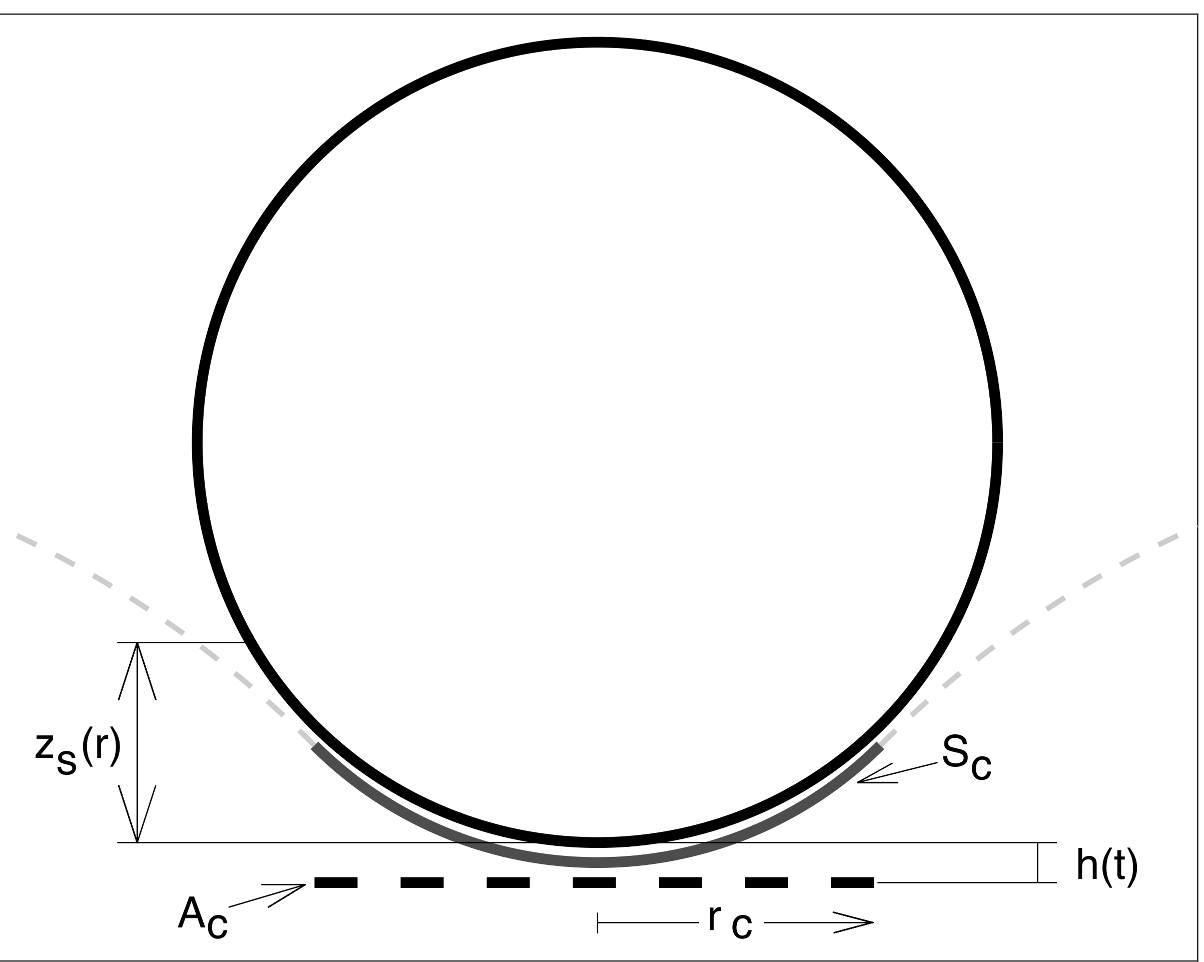

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$$\partial_r \eta(r_c) = \partial_r z_s(r_c).$$

A Non-local Formulation in Physical Space

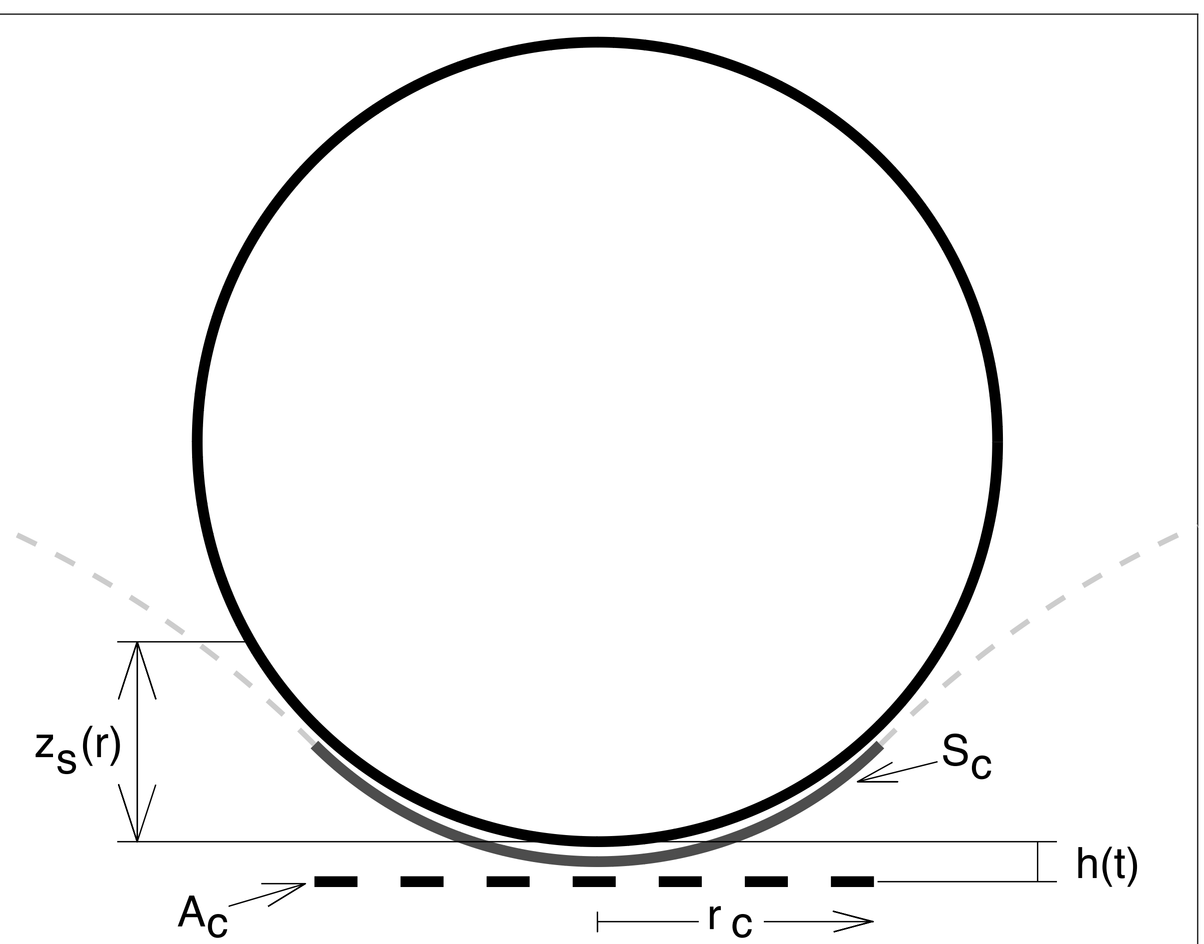
$$\eta_t = \frac{2}{Re}\Delta_H \eta + \phi_z, \qquad z = 0,$$

$$\phi_t = -\frac{1}{Fr}\eta + \frac{1}{We}\,\kappa[\eta] + \frac{2}{Re}\Delta_H \phi - p_s \qquad z = 0;$$

subject to $\eta \to 0$ when $\sqrt{x^2+y^2} \to \infty$

$\phi, \nabla\phi \to 0$ when $\sqrt{x^2+y^2+z^2} \to \infty.$

$$\phi_z(\mathbf{r}) = \frac{1}{2\pi}\lim_{\epsilon\to 0^+}\int_{\mathbb{R}^2\setminus B(\mathbf{r};\epsilon)} \frac{\phi(\mathbf{r}) - \phi(\mathbf{s})}{|\mathbf{r}-\mathbf{s}|^3}\, dA(\mathbf{s})$$



$$h_{tt} = -\frac{1}{Fr}h - Dh_t + \frac{1}{M}\int_{r\le r_c} p_s\, dA$$

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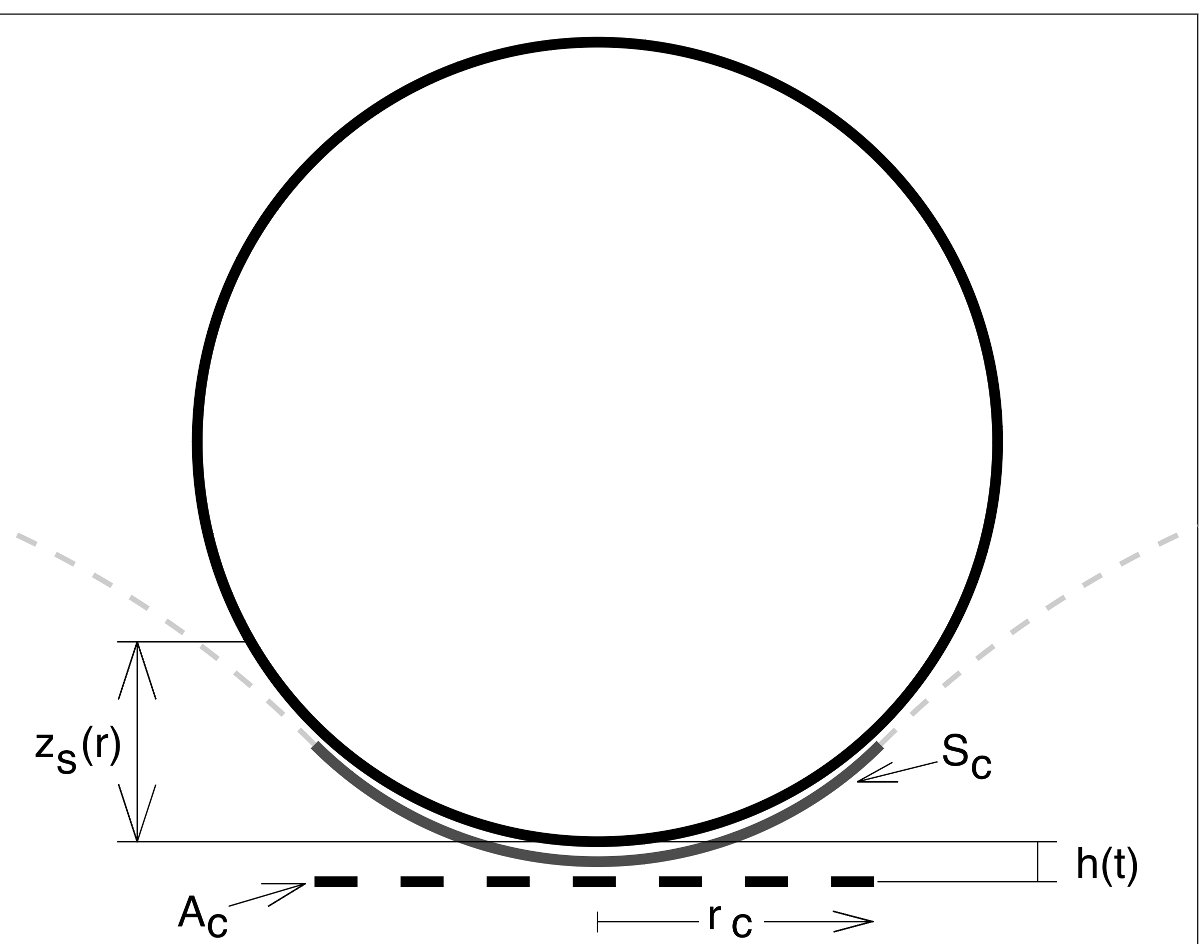

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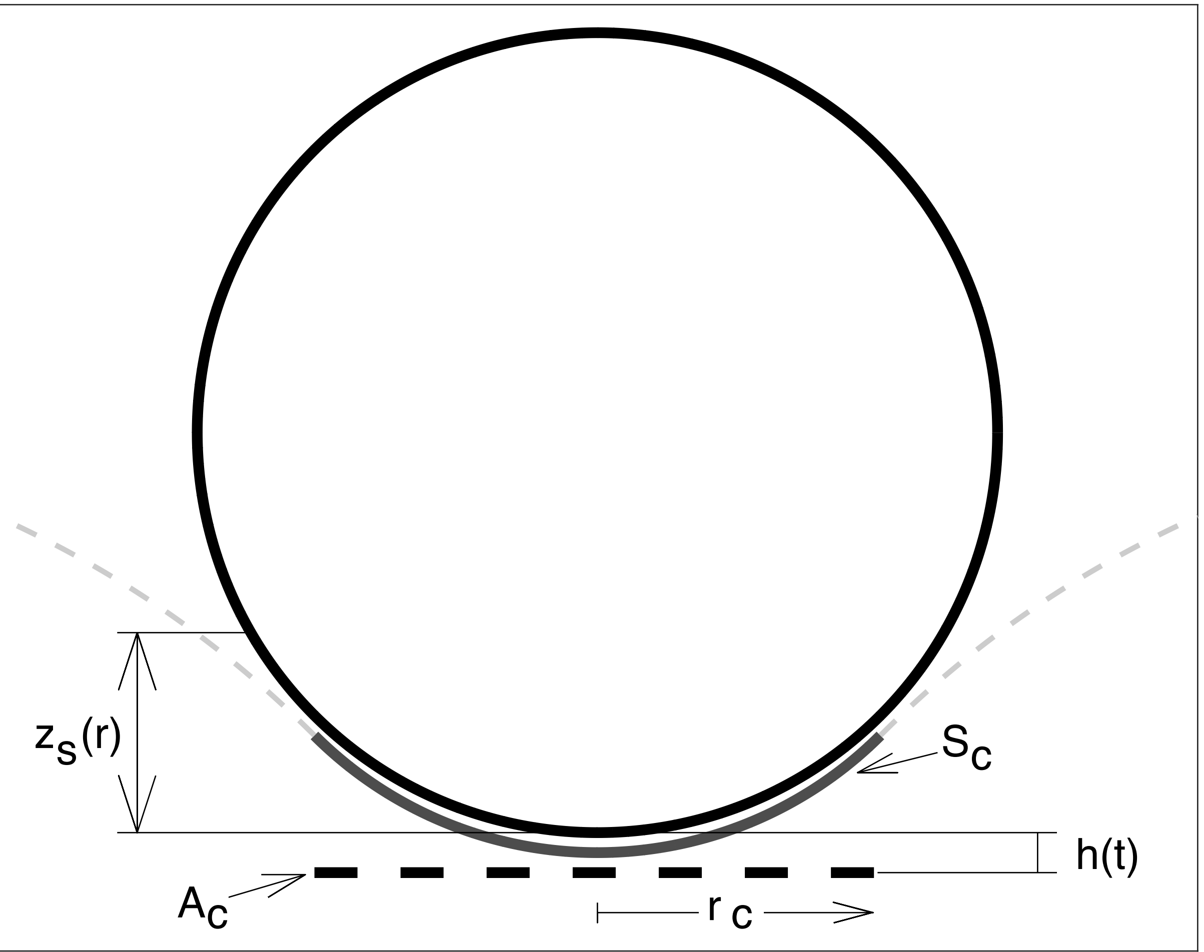

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$$\boldsymbol{Q} = \begin{bmatrix} \left(\boldsymbol{I} - \frac{2\delta t}{Re}\Delta_H\right) & -\delta t N & 0 & 0 & 0 \\ \delta t \left(\frac{1}{Fr}\boldsymbol{I} - \frac{1}{We}\Delta_H\right) & \left(\boldsymbol{I} - \frac{2\delta t}{Re}\Delta_H\right) & \delta t \boldsymbol{I} & 0 & 0 \\ 0 & 0 & -\delta t \frac{A}{M} & (1 + \delta t D) & 0 \\ 0 & 0 & 0 & -\delta t & 1 \end{bmatrix},$$

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Superindex k

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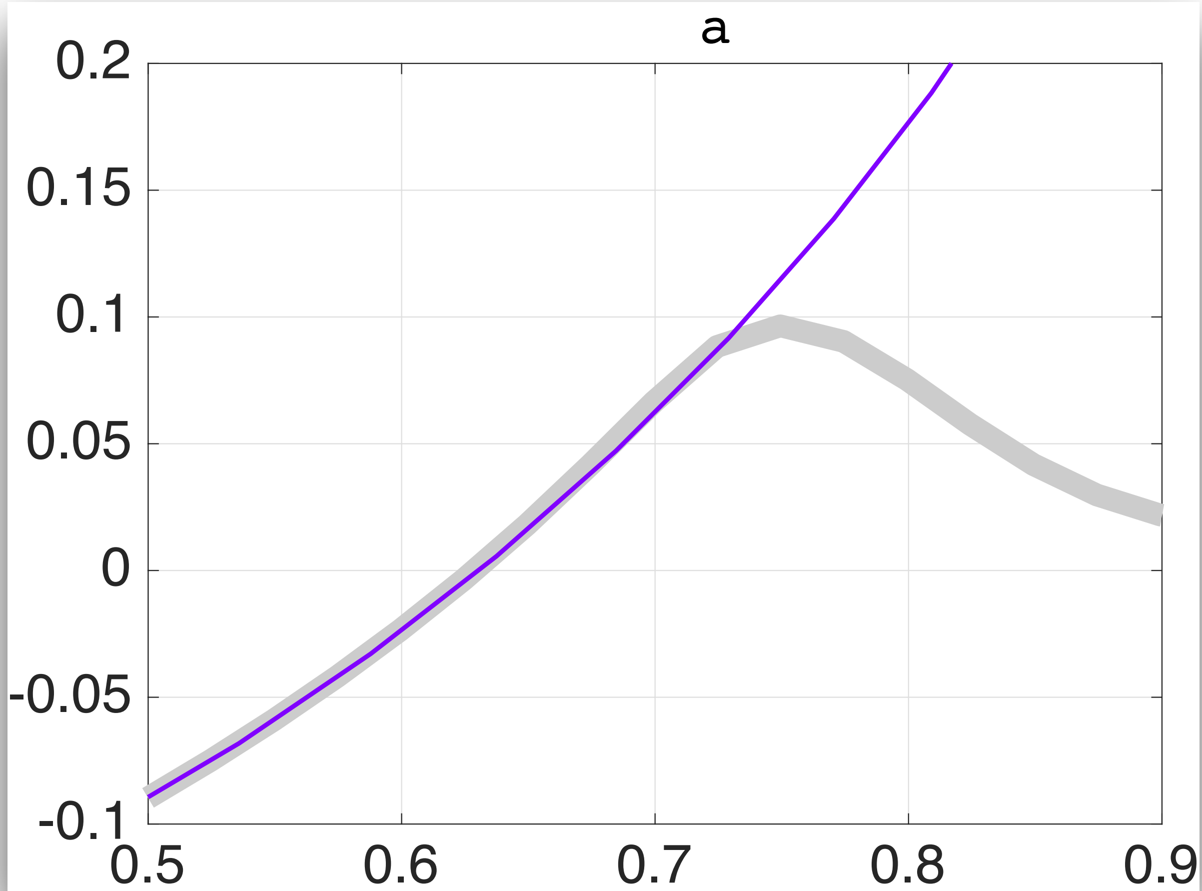

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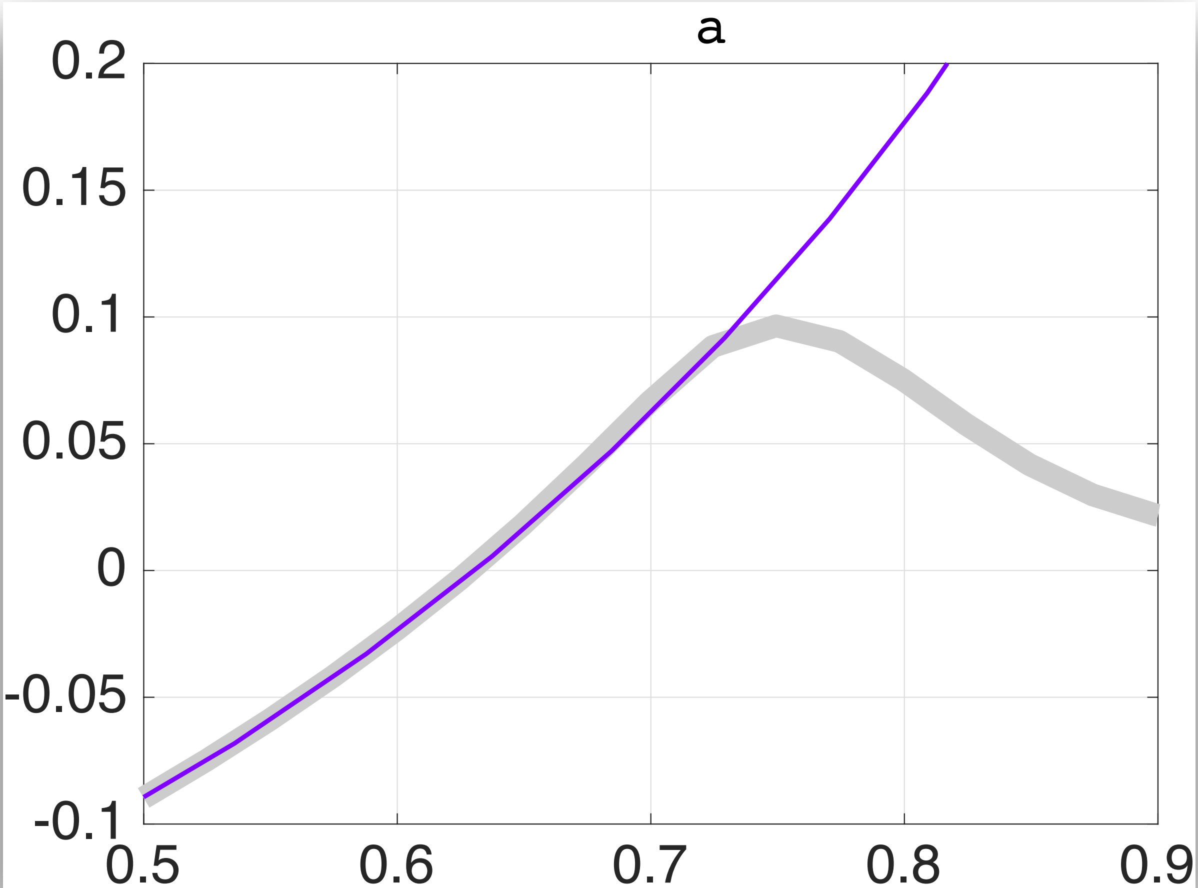

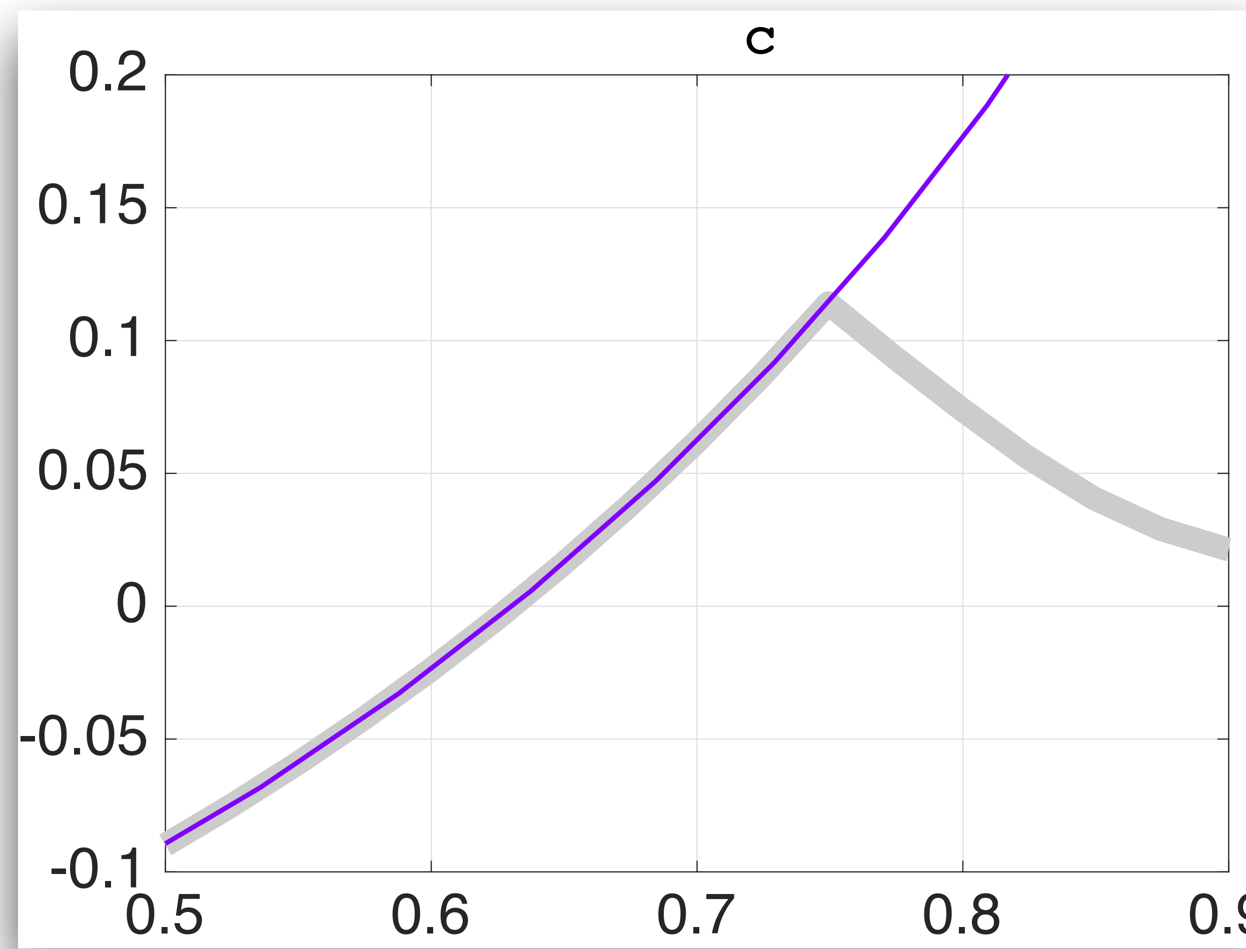

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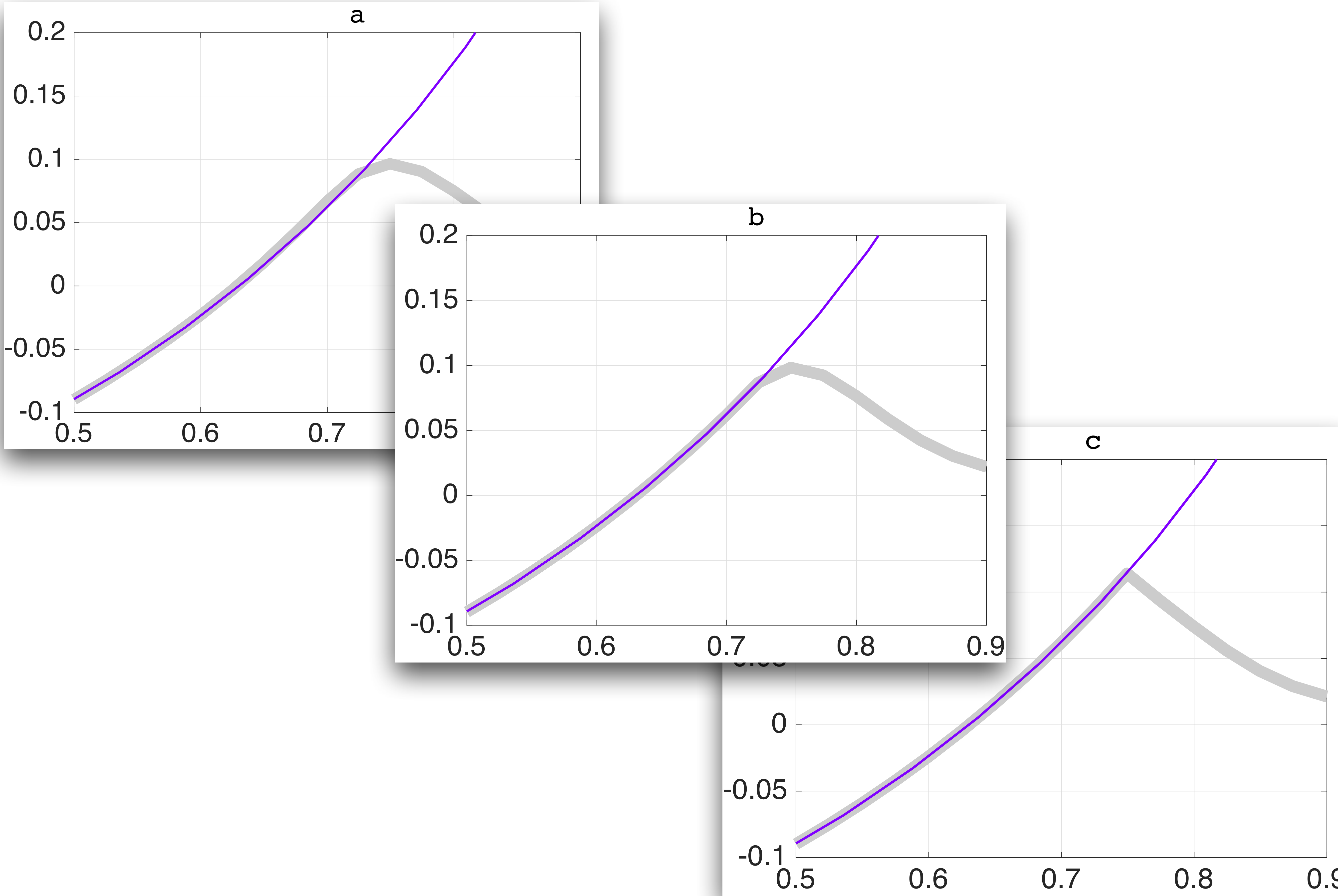
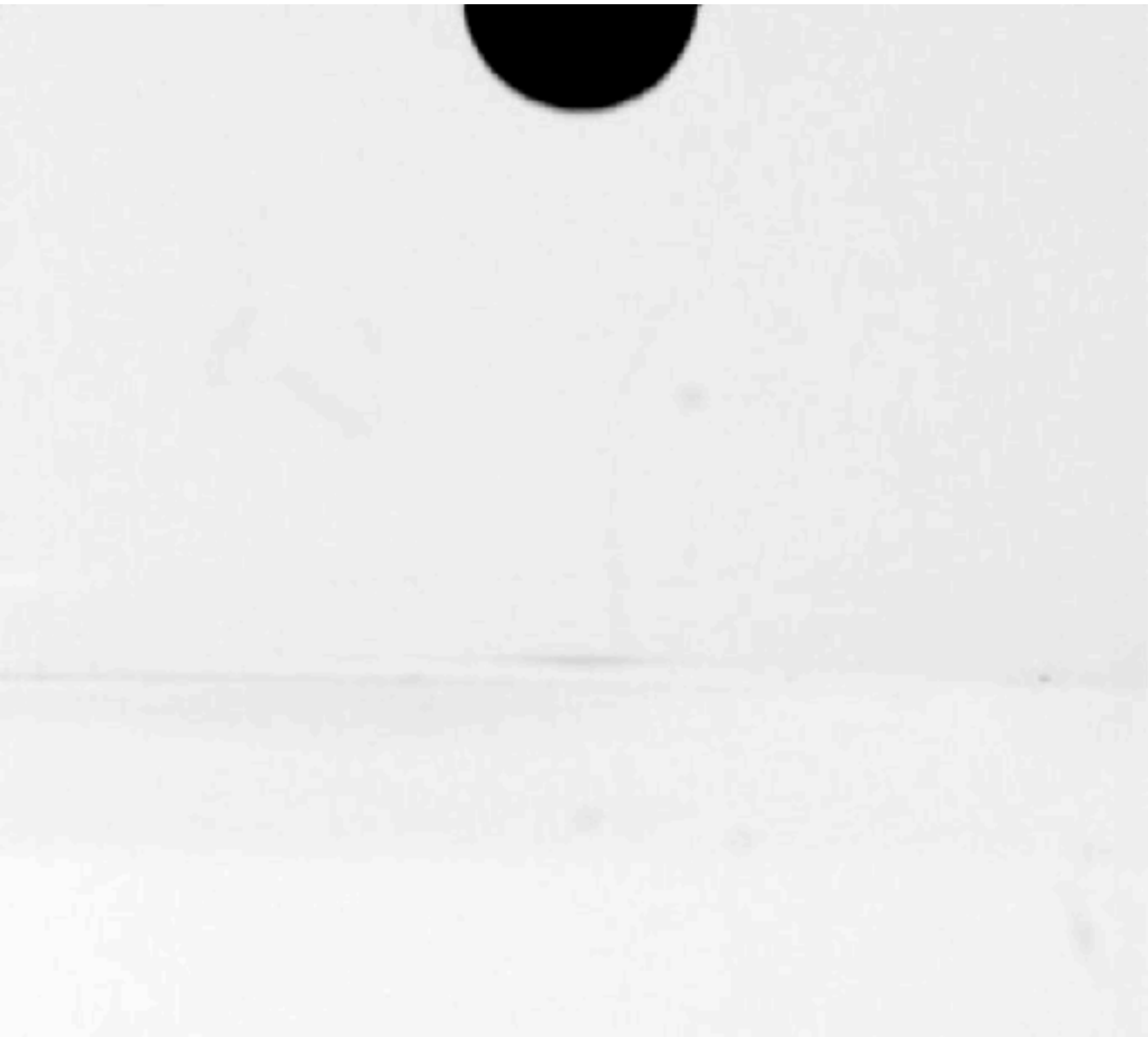
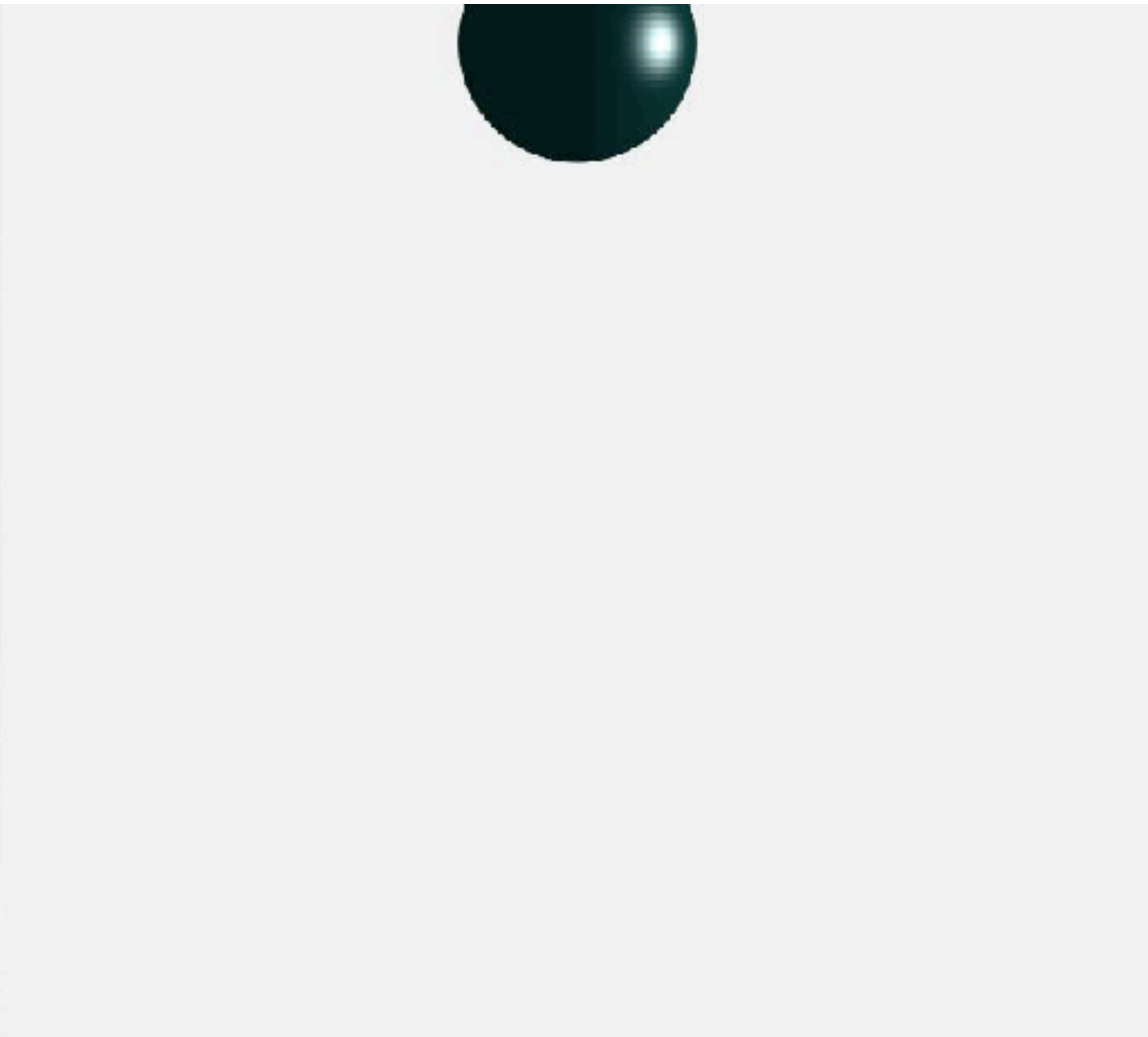
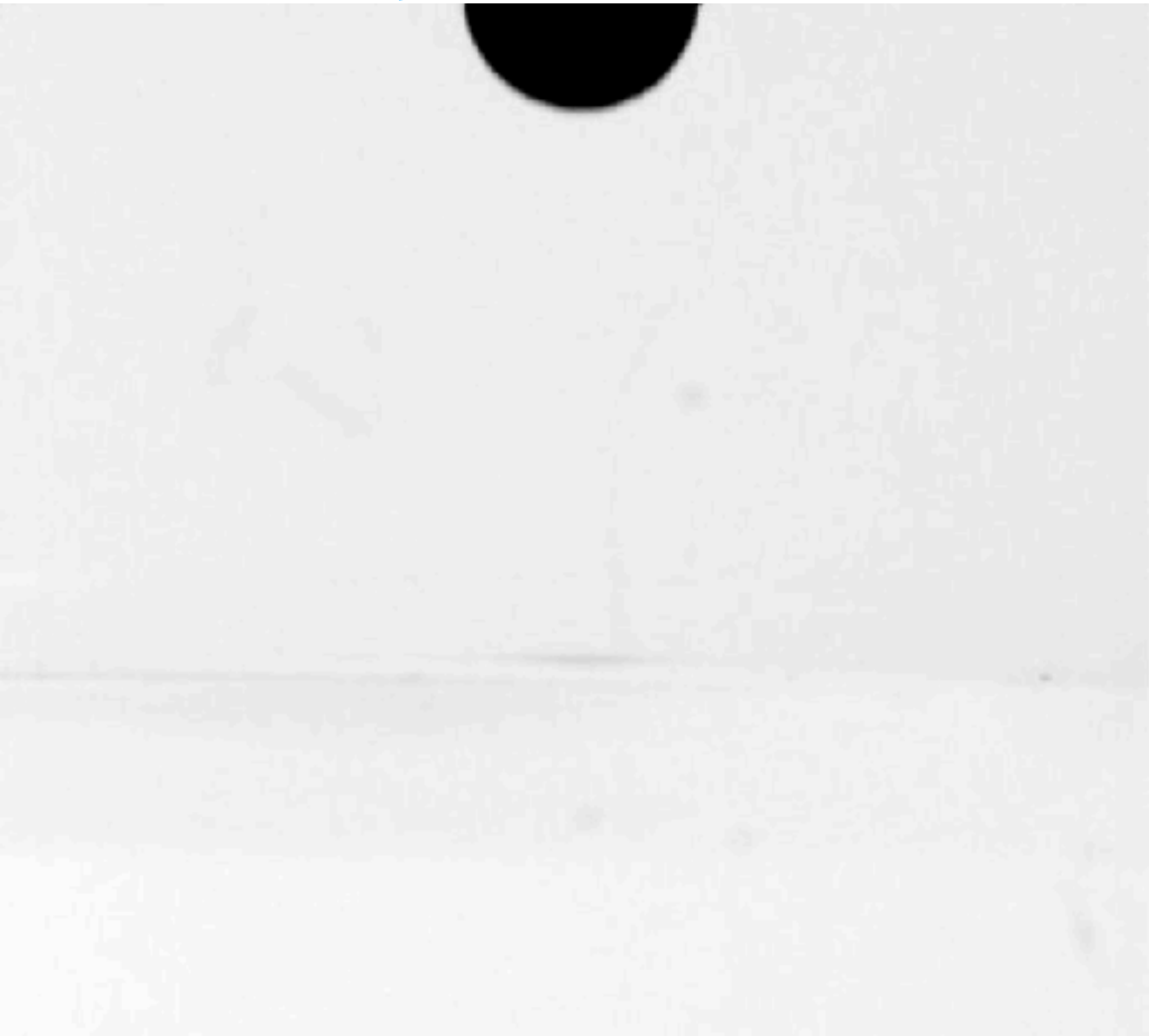
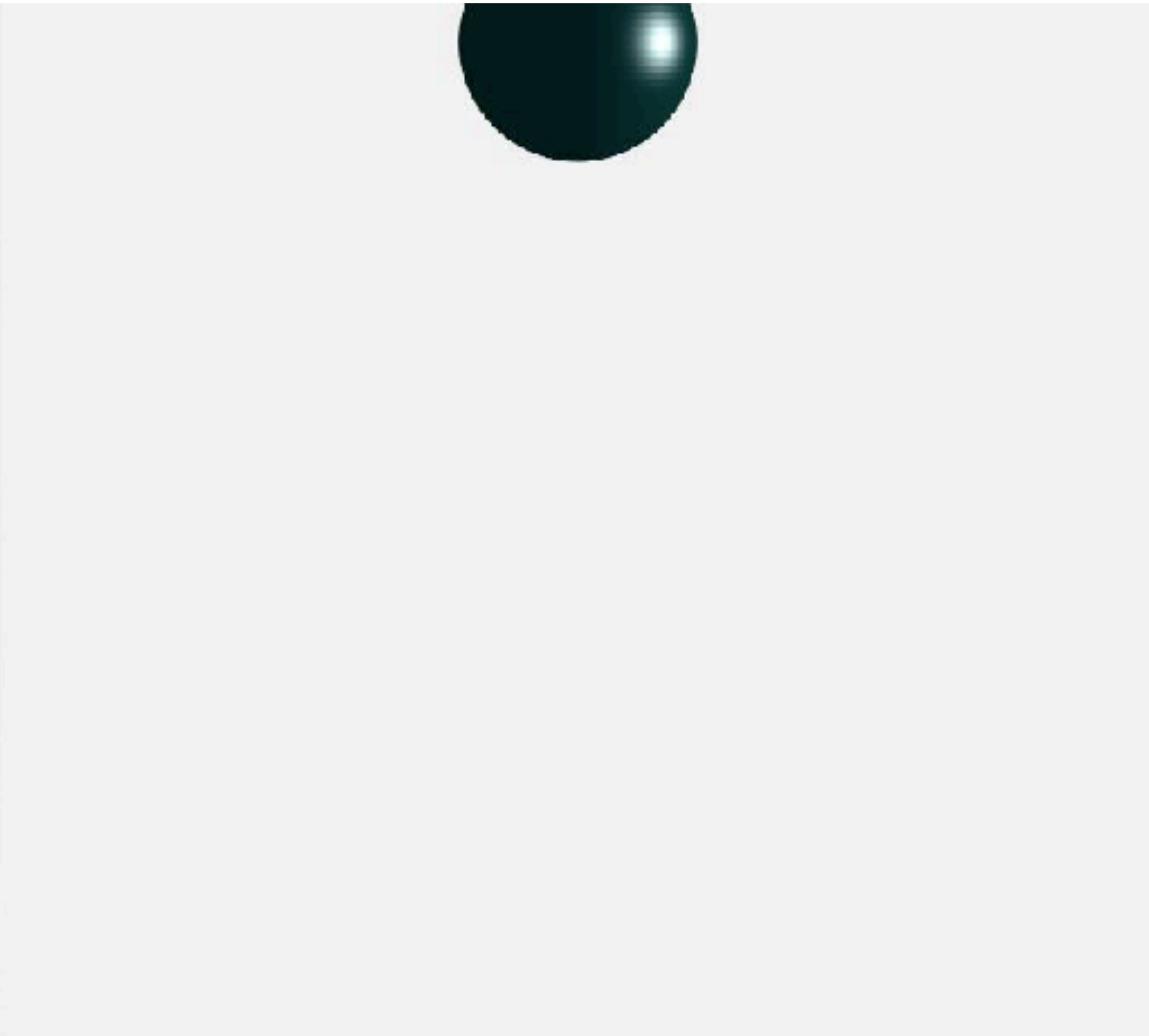


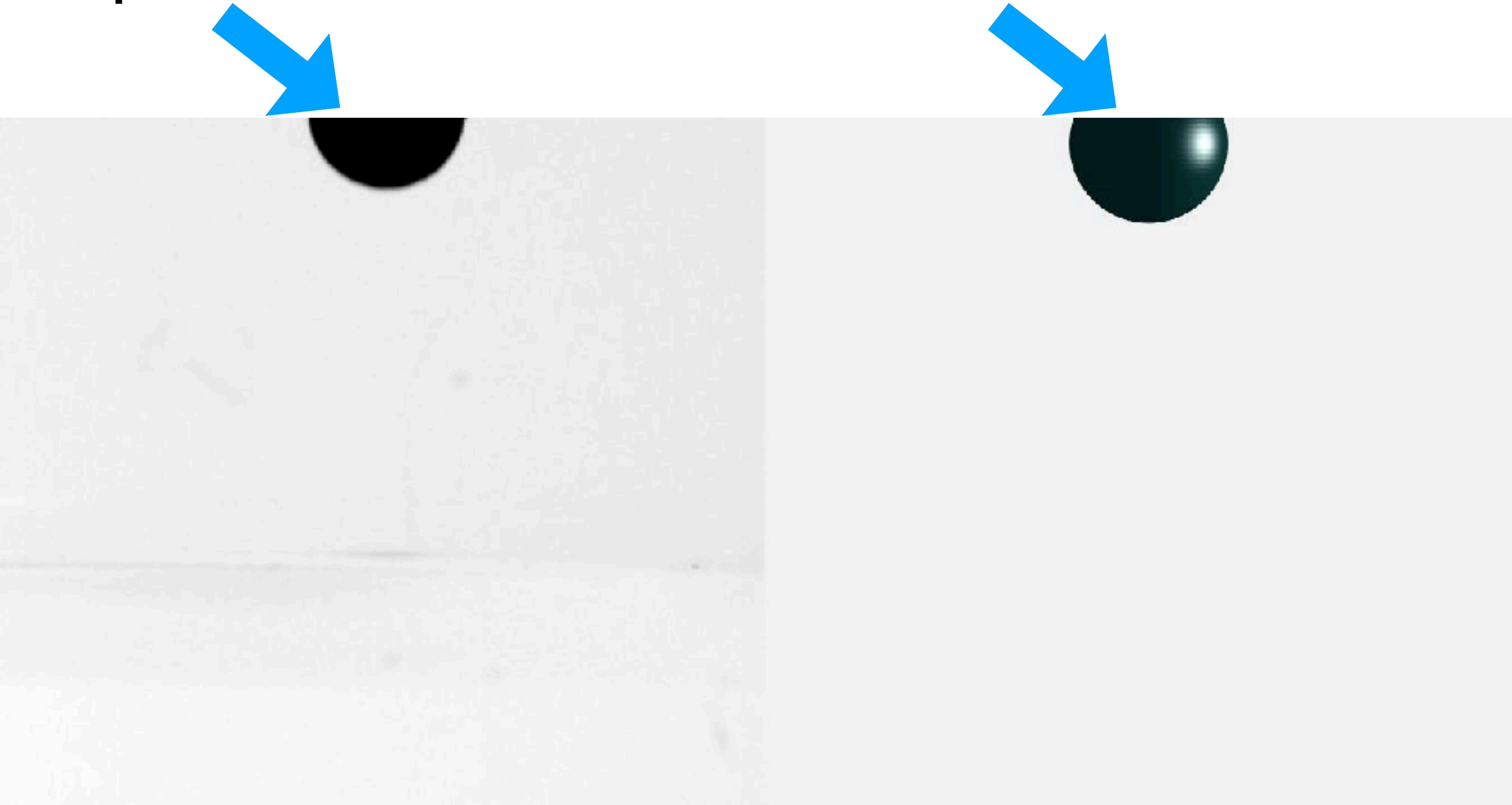
 

Experiment

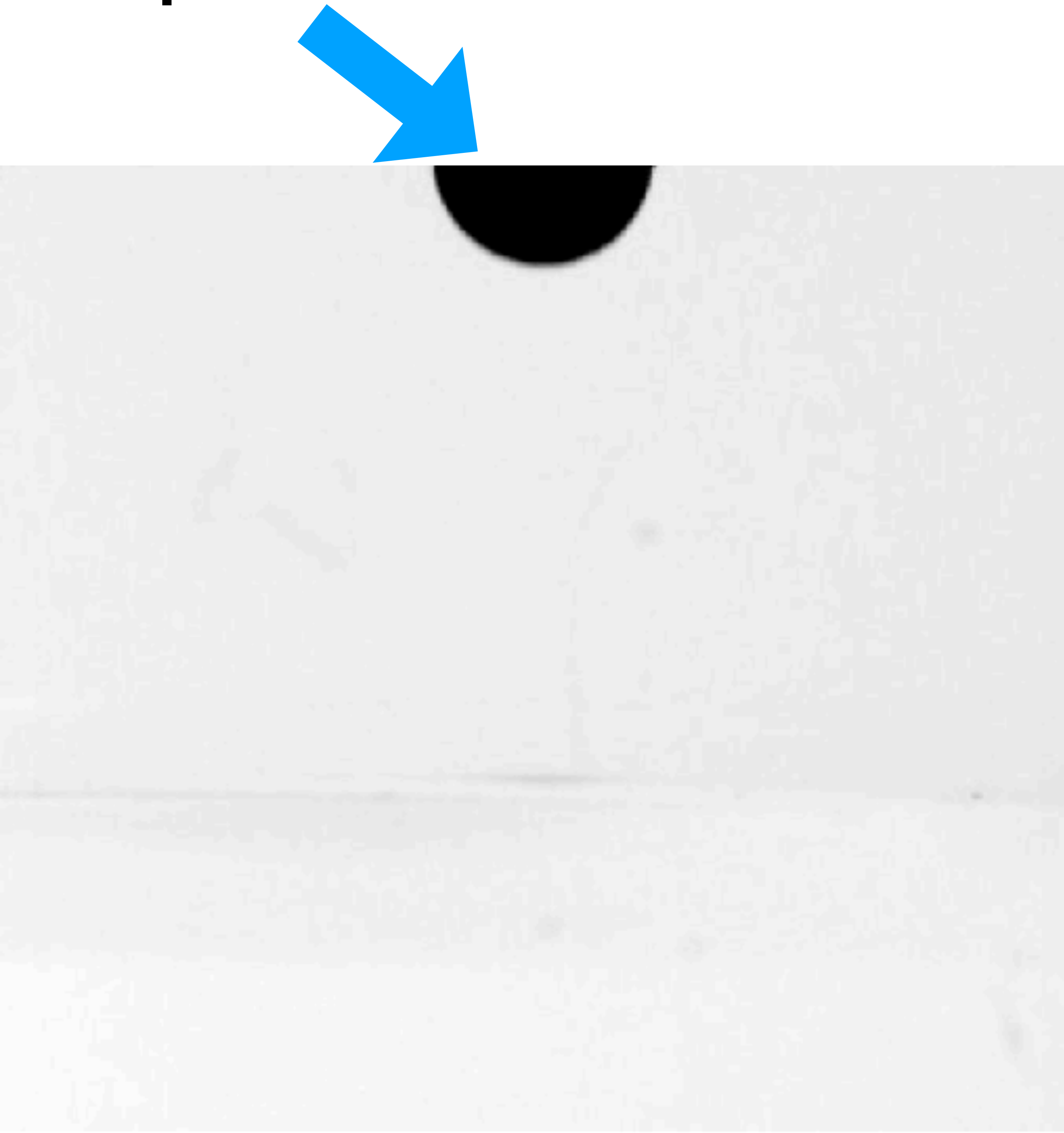




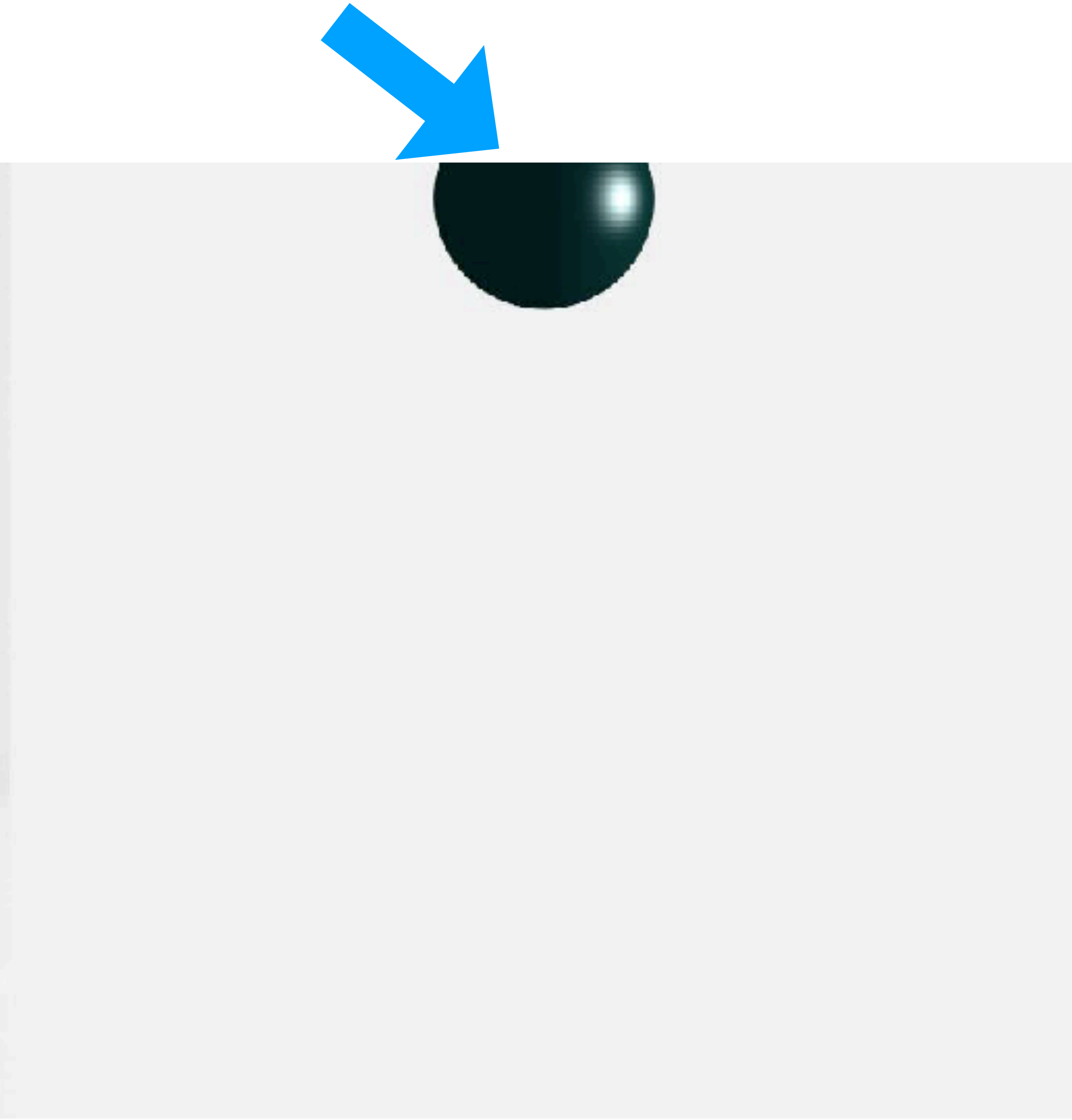
Experiment **Simulation**



Experiment



Simulation



Experiment by Dan Harris (Brown)

Droplets on a Shaking Free Surface

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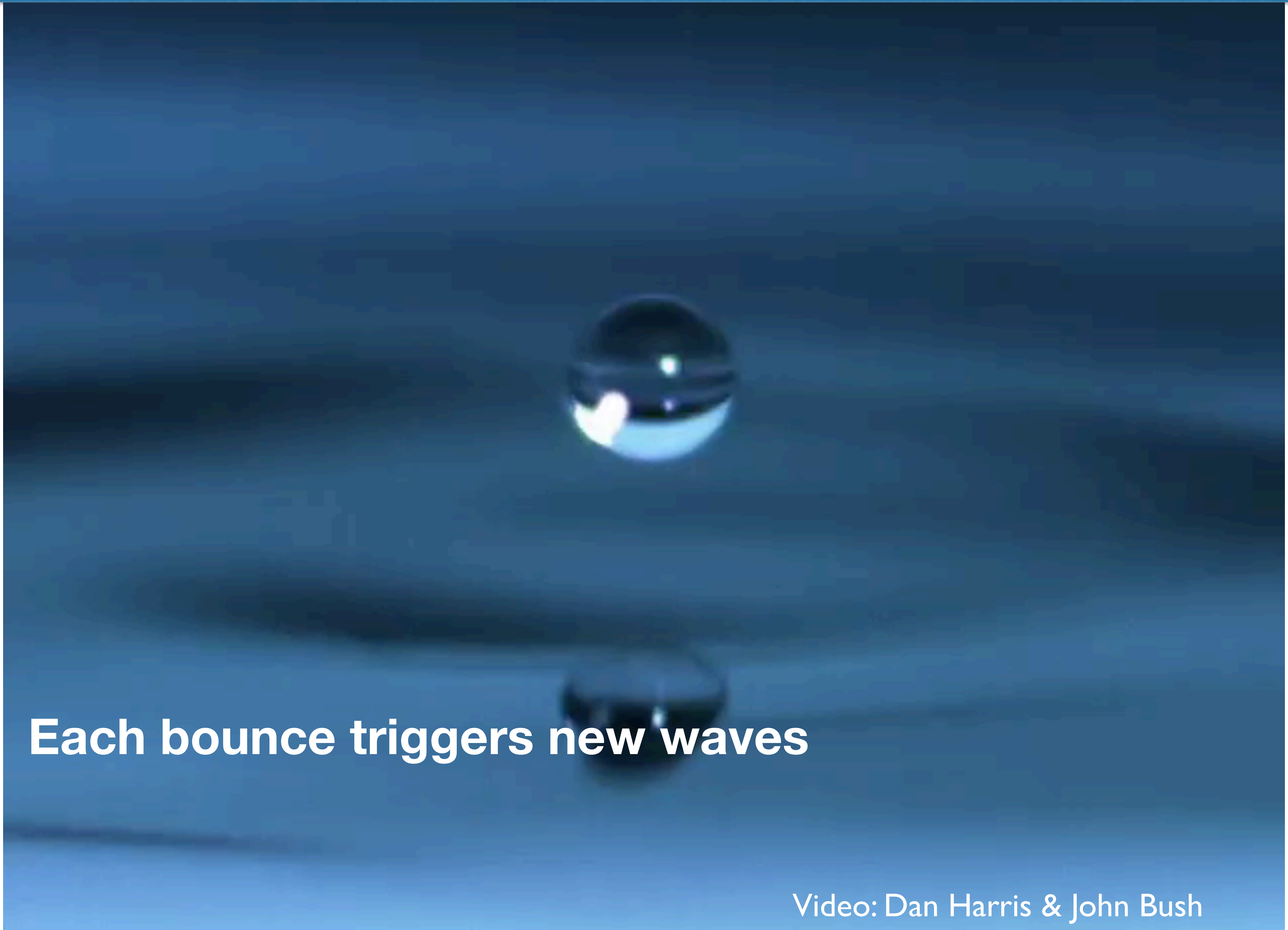
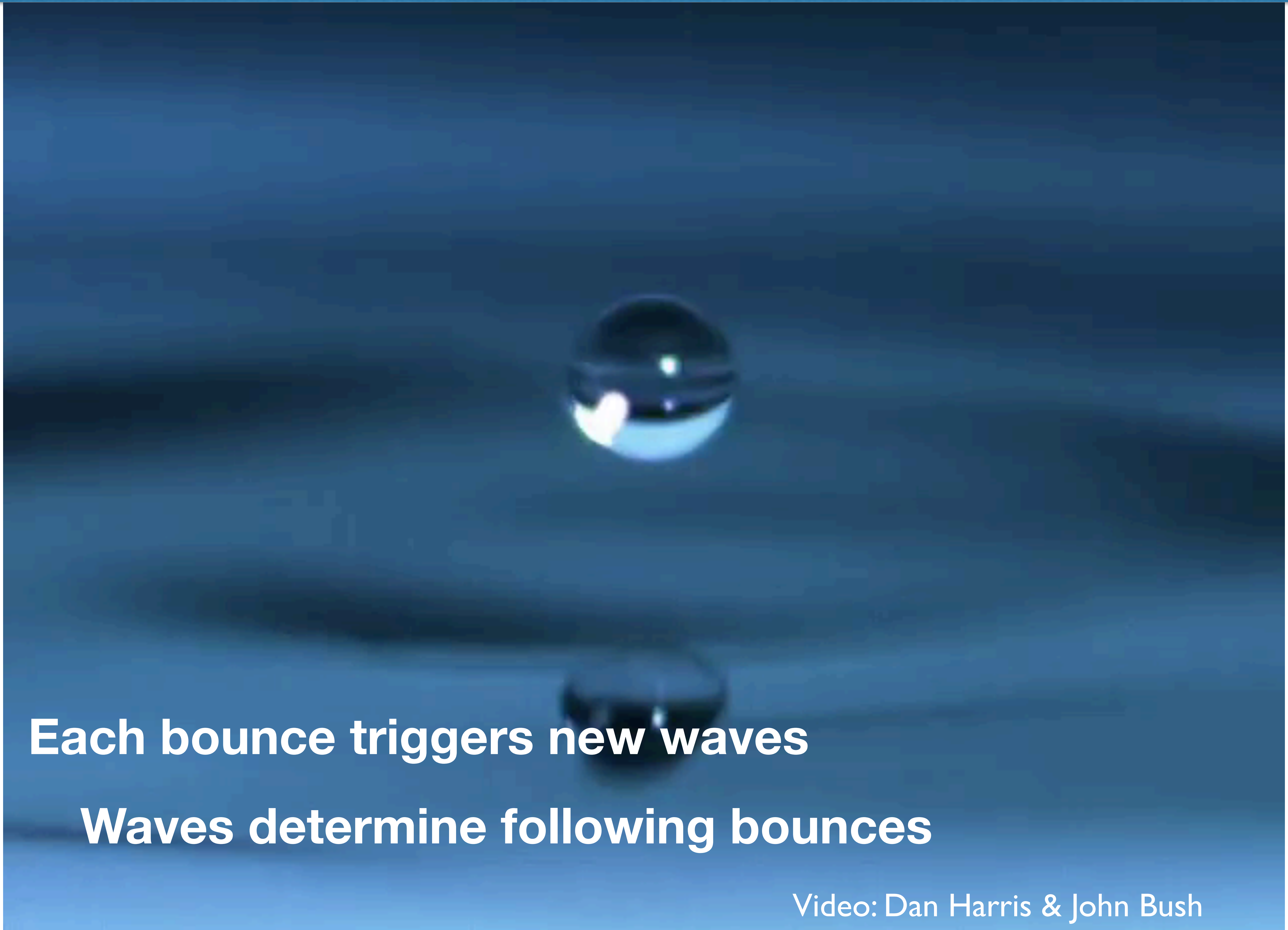


Droplets on a Shaking Free Surface



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Droplets on a Shaking Free Surface

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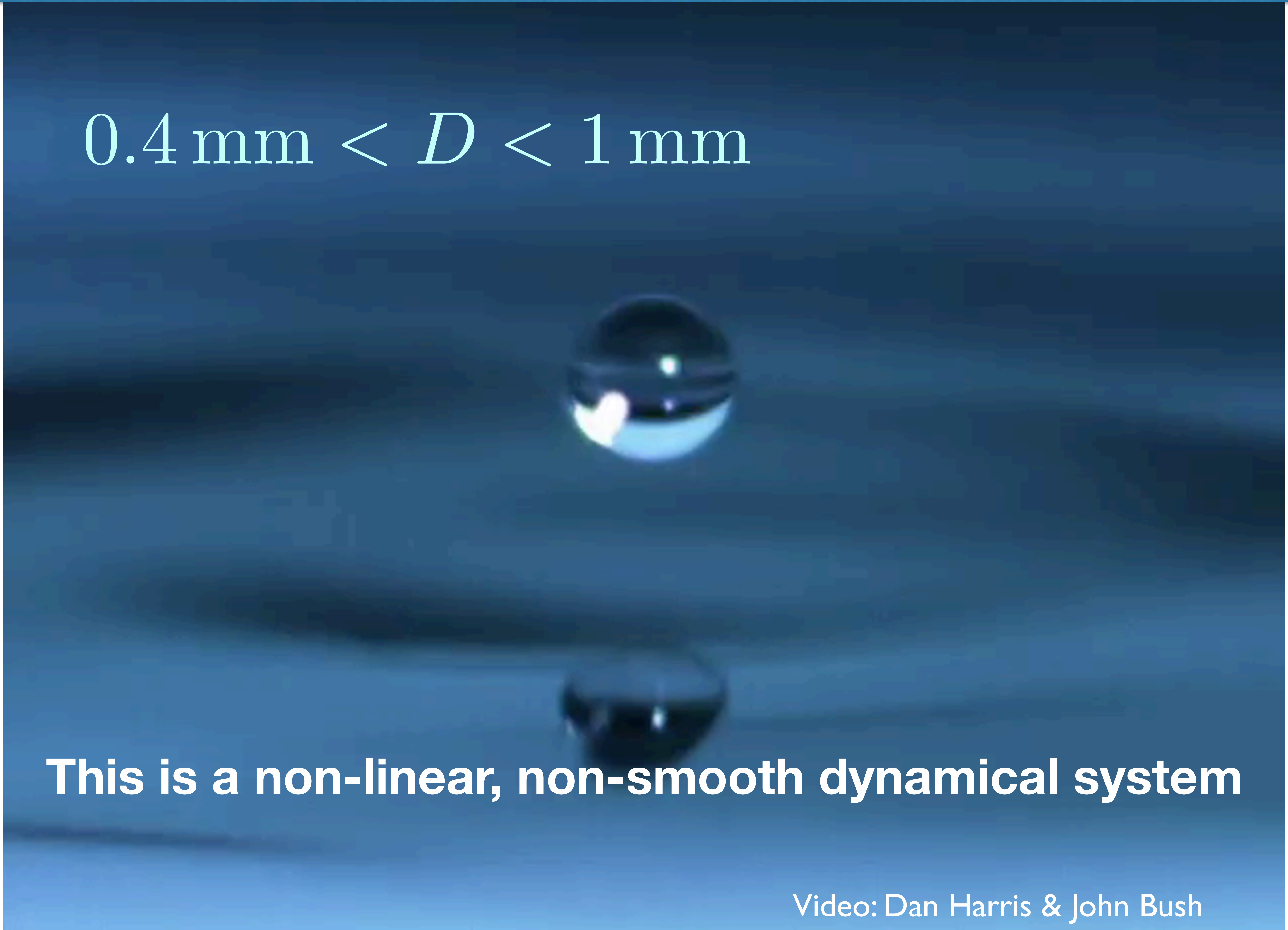
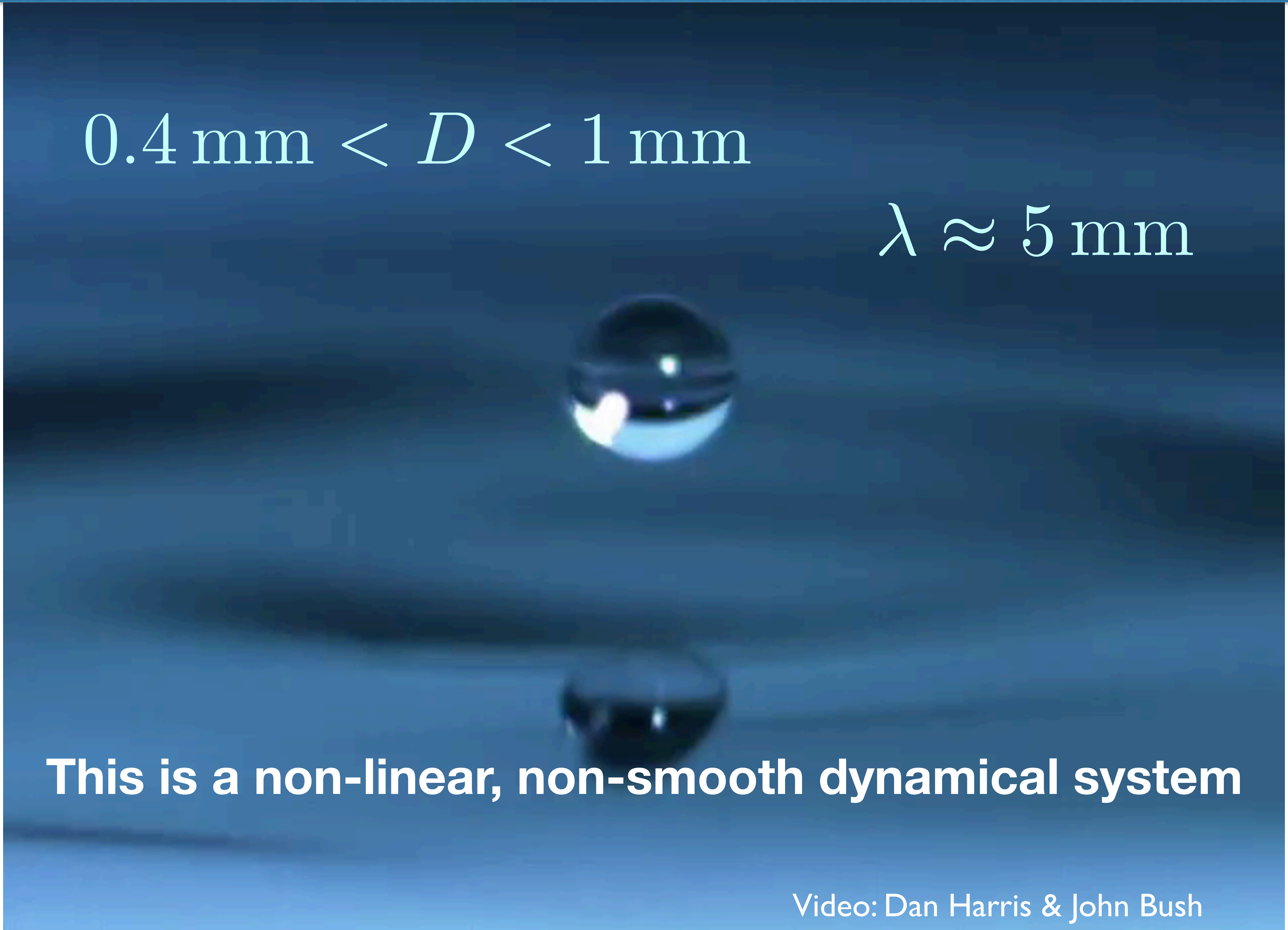
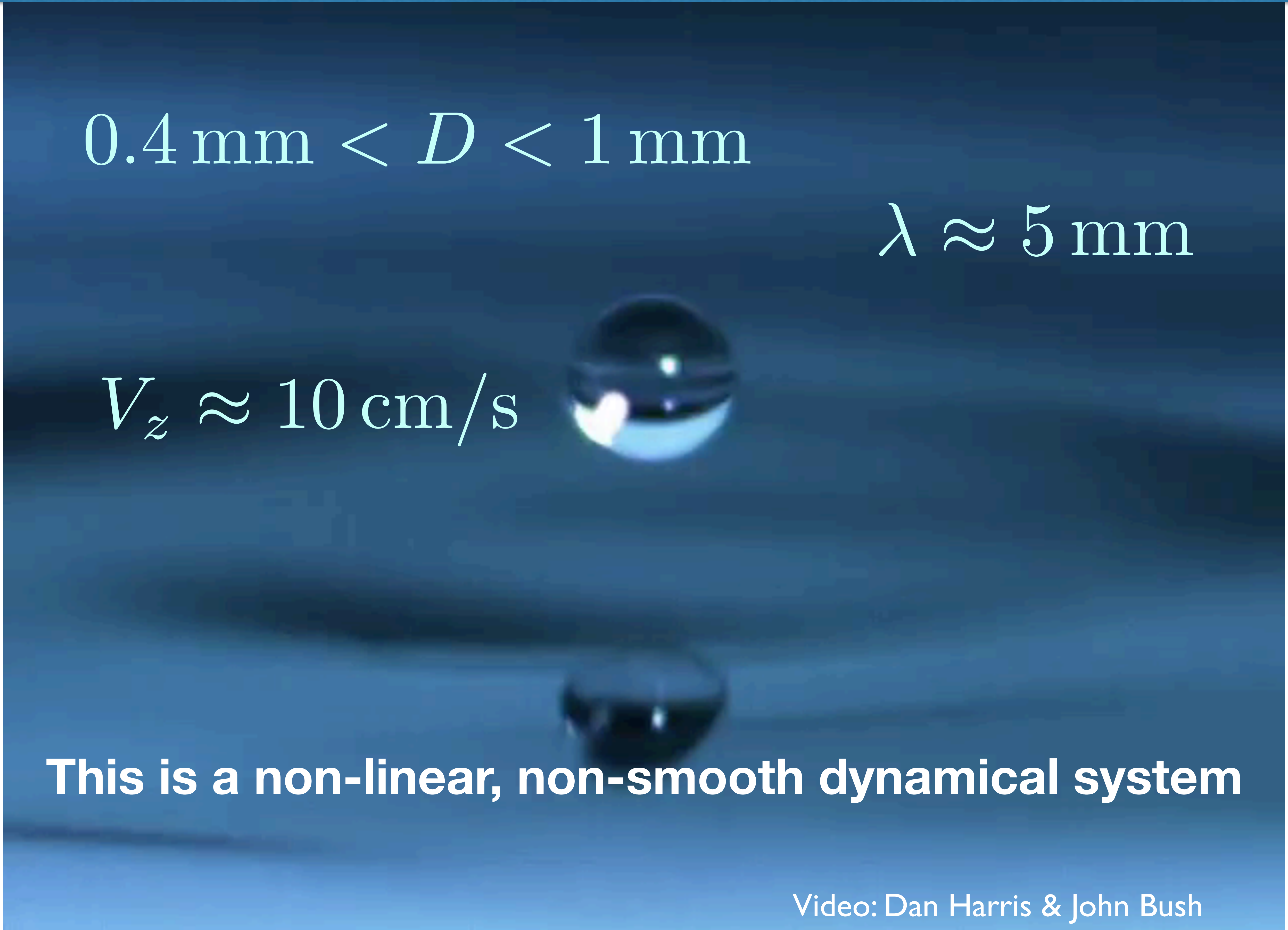


Droplets on a Shaking Free Surface



Droplets on a Shaking Free Surface



$$0.4\,\mathrm{mm} < D < 1\,\mathrm{mm}$$

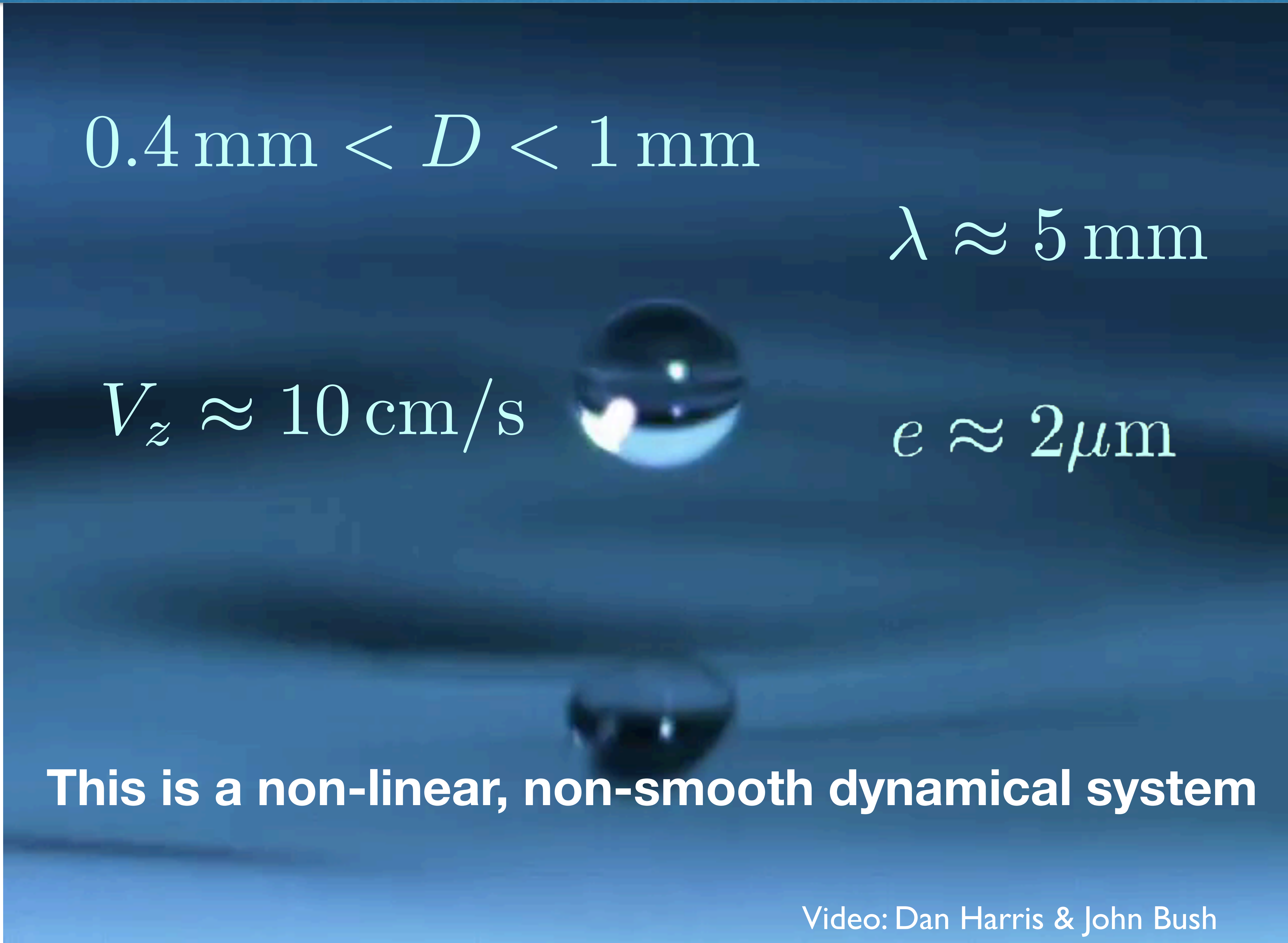
$$\lambda \approx 5\,\mathrm{mm}$$

$$V_z \approx 10\,\mathrm{cm/s}$$

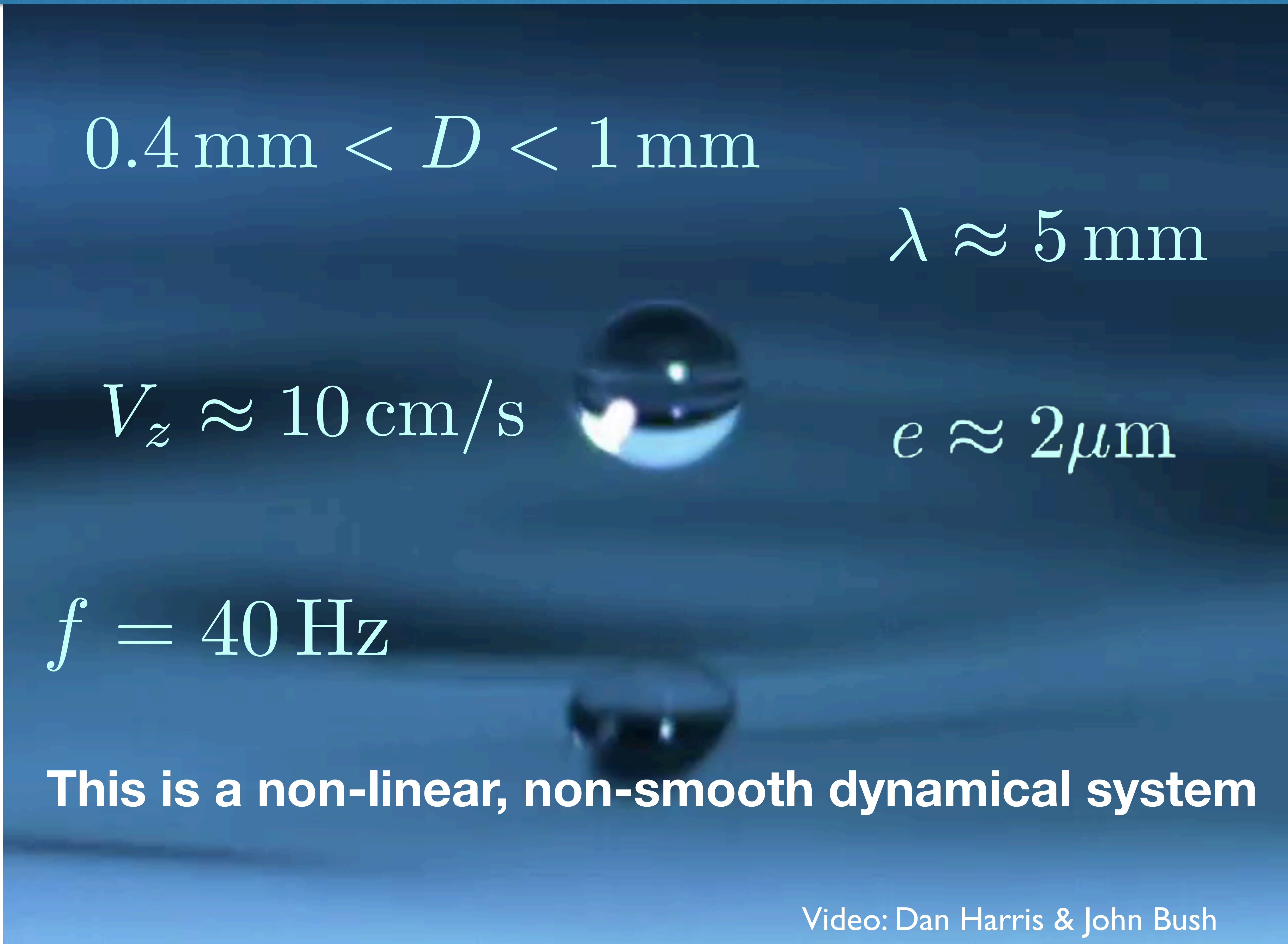
This is a non-linear, non-smooth dynamical system

Video: Dan Harris & John Bush

Droplets on a Shaking Free Surface



Droplets on a Shaking Free Surface



$$0.4\,\mathrm{mm} < D < 1\,\mathrm{mm}$$

$$\lambda \approx 5\,\mathrm{mm}$$

$$V_z \approx 10\,\mathrm{cm/s}$$

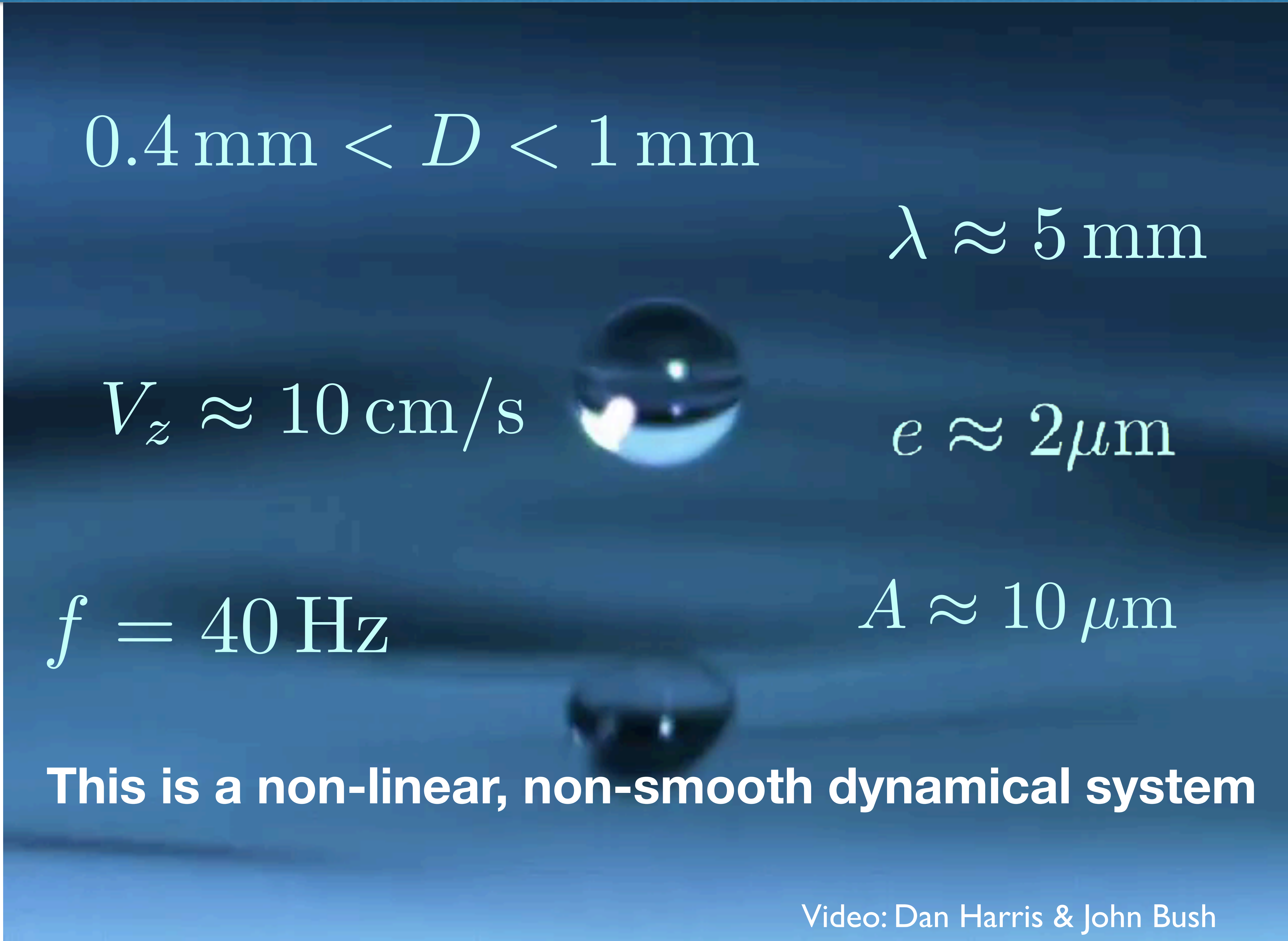
$$e \approx 2\mu\mathrm{m}$$

$$f = 40\,\mathrm{Hz}$$

This is a non-linear, non-smooth dynamical system

Video: Dan Harris & John Bush

Droplets on a Shaking Free Surface



$$0.4\,\text{mm} < D < 1\,\text{mm}$$

$$\lambda \approx 5\,\text{mm}$$

$$V_z \approx 10\,\text{cm/s}$$

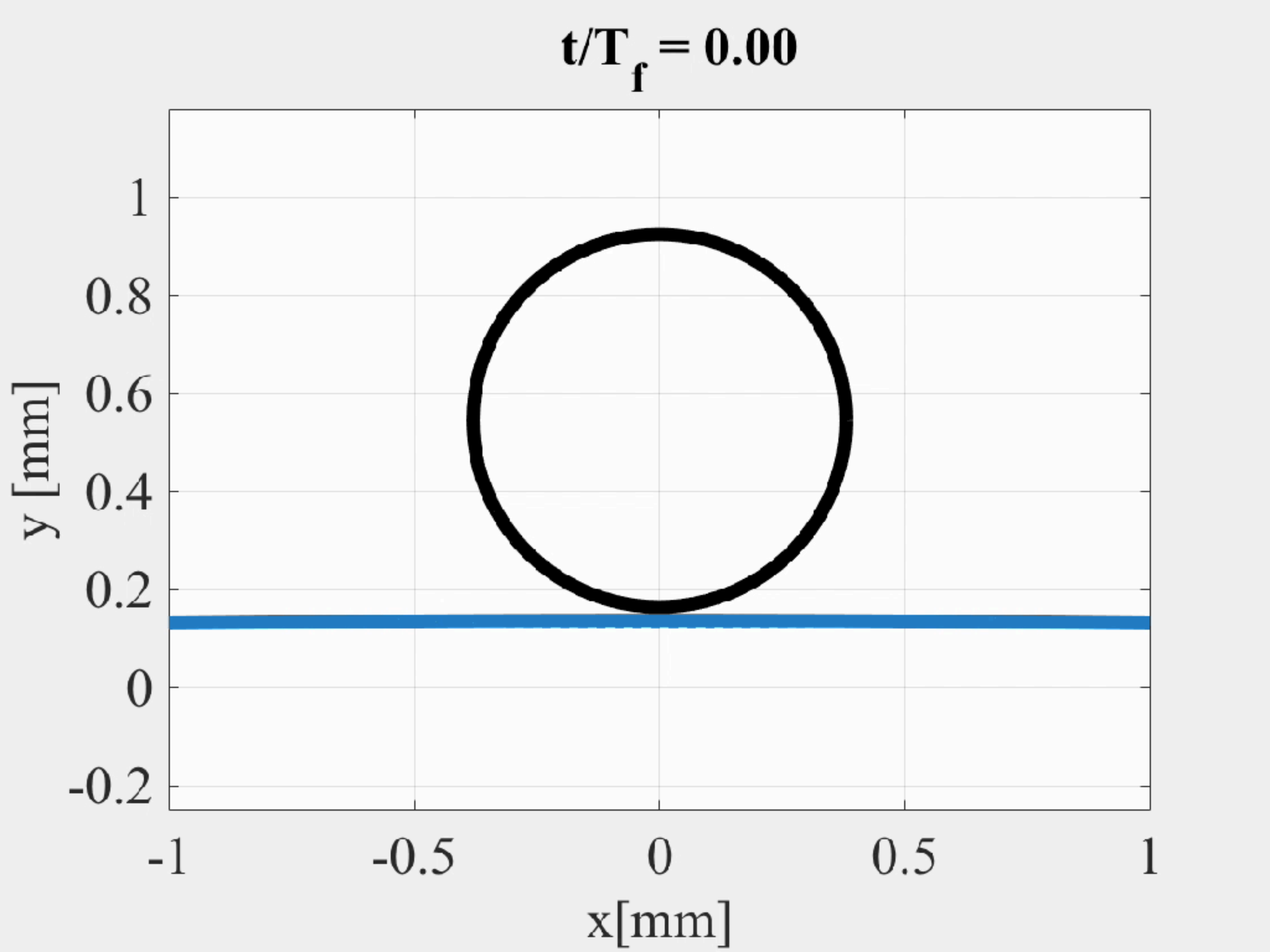
$$e \approx 2\mu\text{m}$$

$$f = 40\,\text{Hz}$$

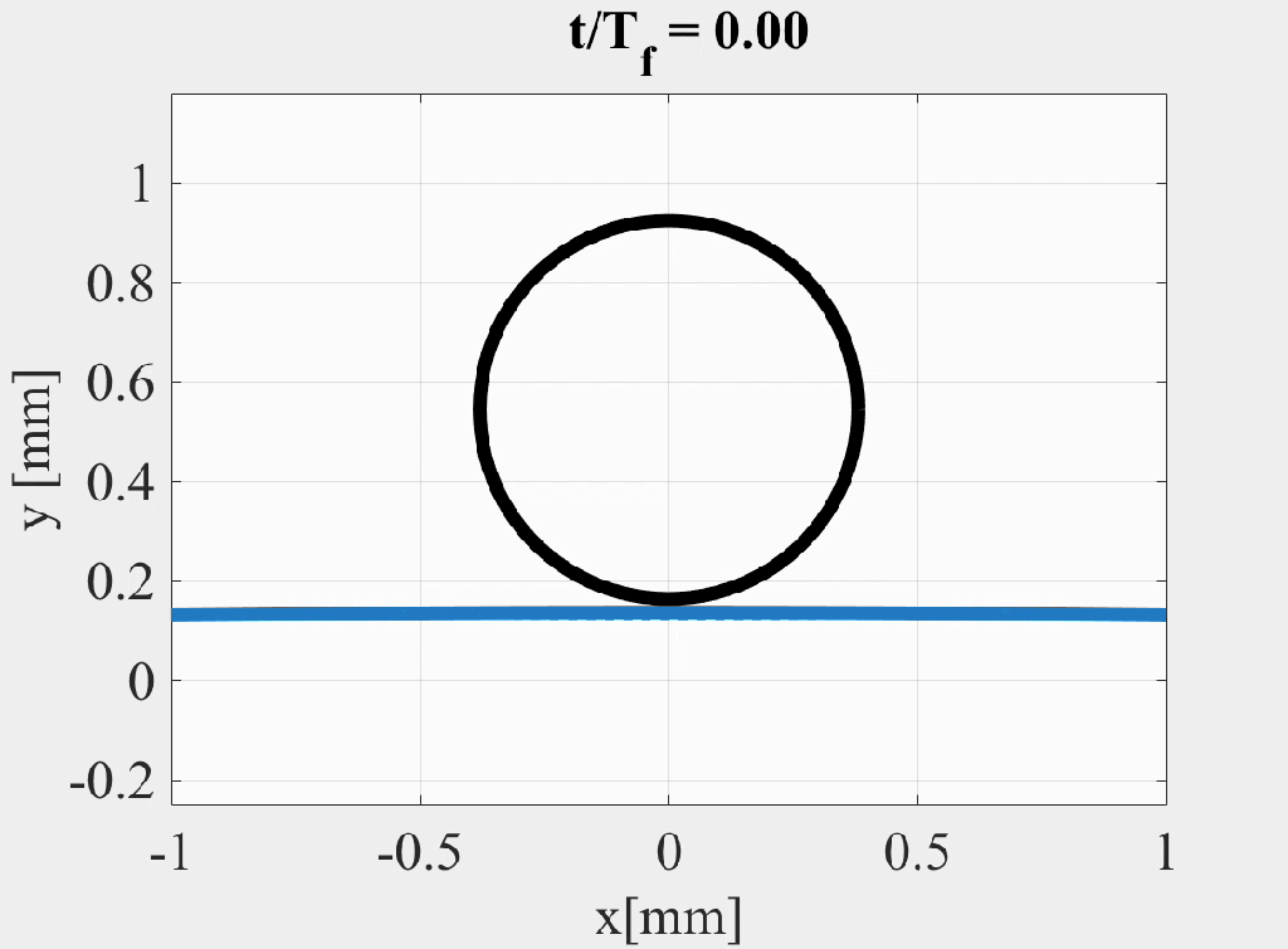
$$A \approx 10\,\mu\text{m}$$

This is a non-linear, non-smooth dynamical system

Video: Dan Harris & John Bush



t/T_f = 0.00
1
0.8
0.6
0.4
0.2
0
-0.2
y [mm]
-1
-0.5
0
0.5
1
x[mm]



t/T_f = 0.00
y [mm]
x[mm]
1
0.8
0.6
0.4
0.2
0
-0.2
-1
-0.5
0
0.5
1

J. Fluid Mech. (2017), *vol.* 826, *pp.* 97–127.

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Non-wetting impact of a sphere onto a bath and its application to bouncing droplets

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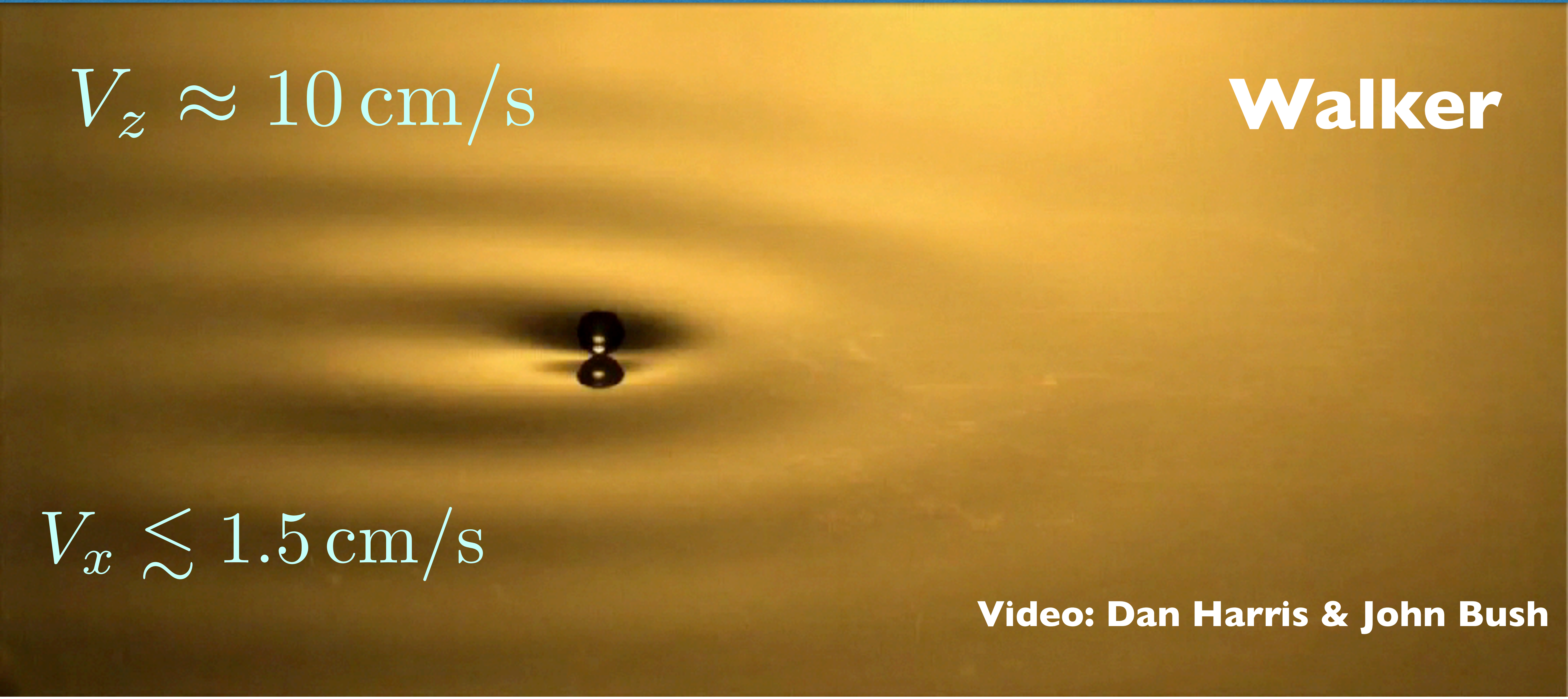
Droplets Walking on a Free Surface

Droplets Walking on a Free Surface

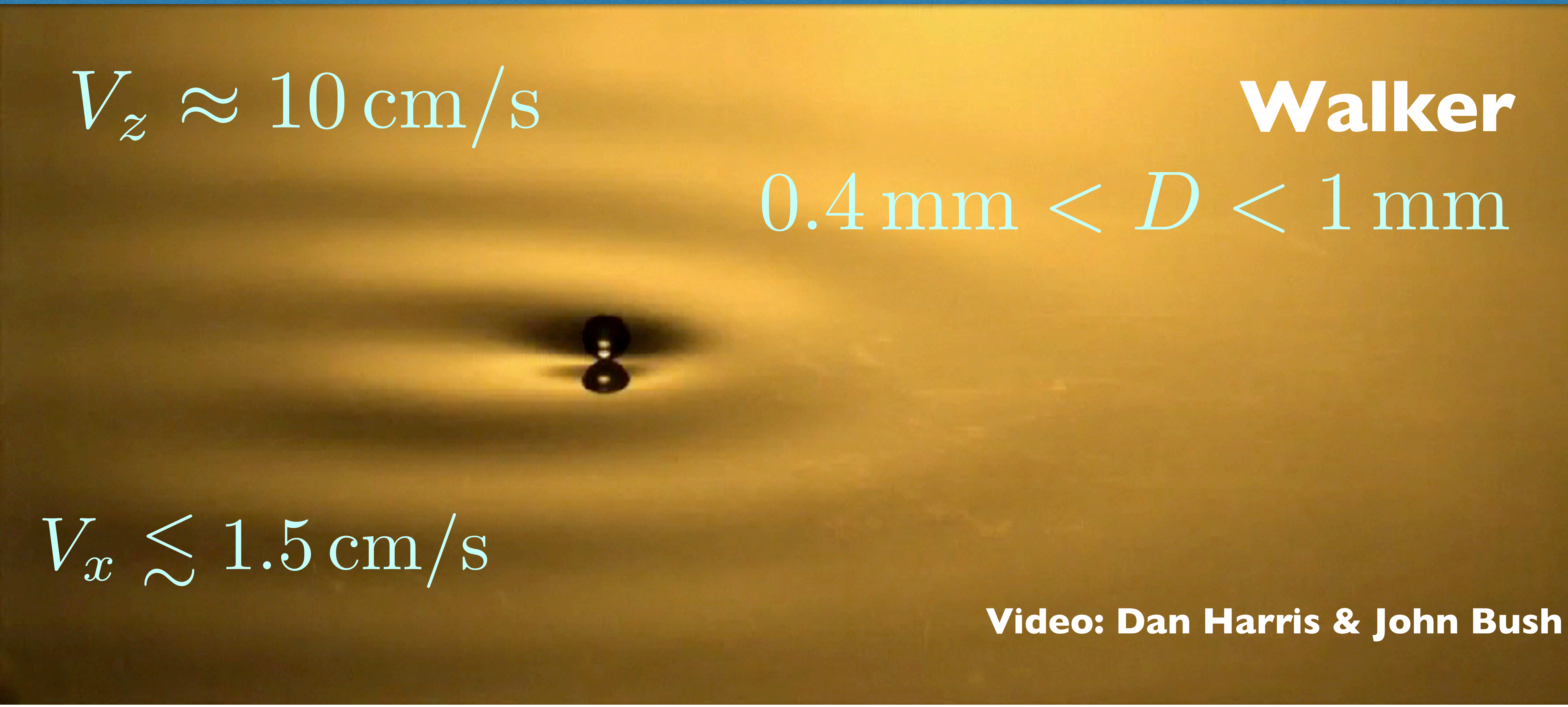
Droplets Walking on a Free Surface

Droplets Walking on a Free Surface

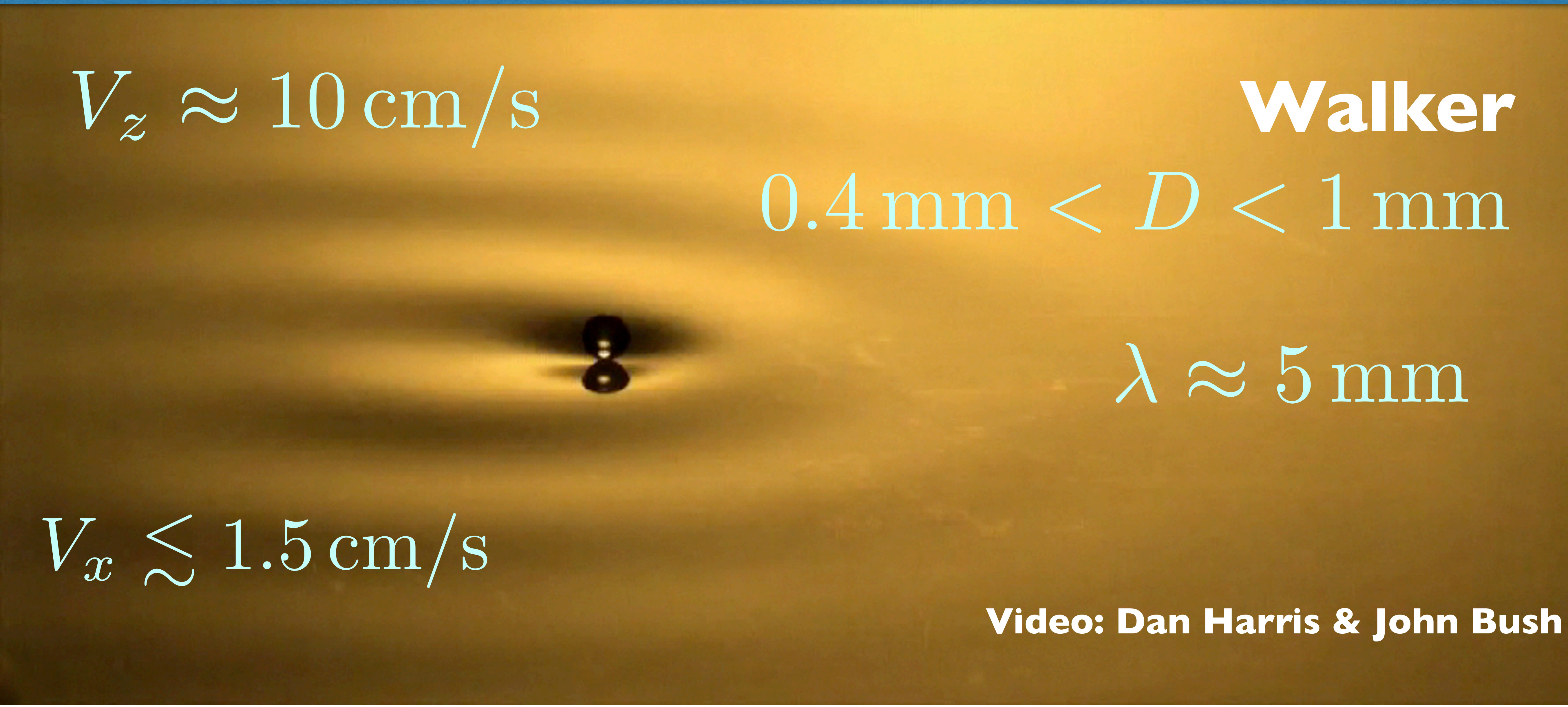
Droplets Walking on a Free Surface



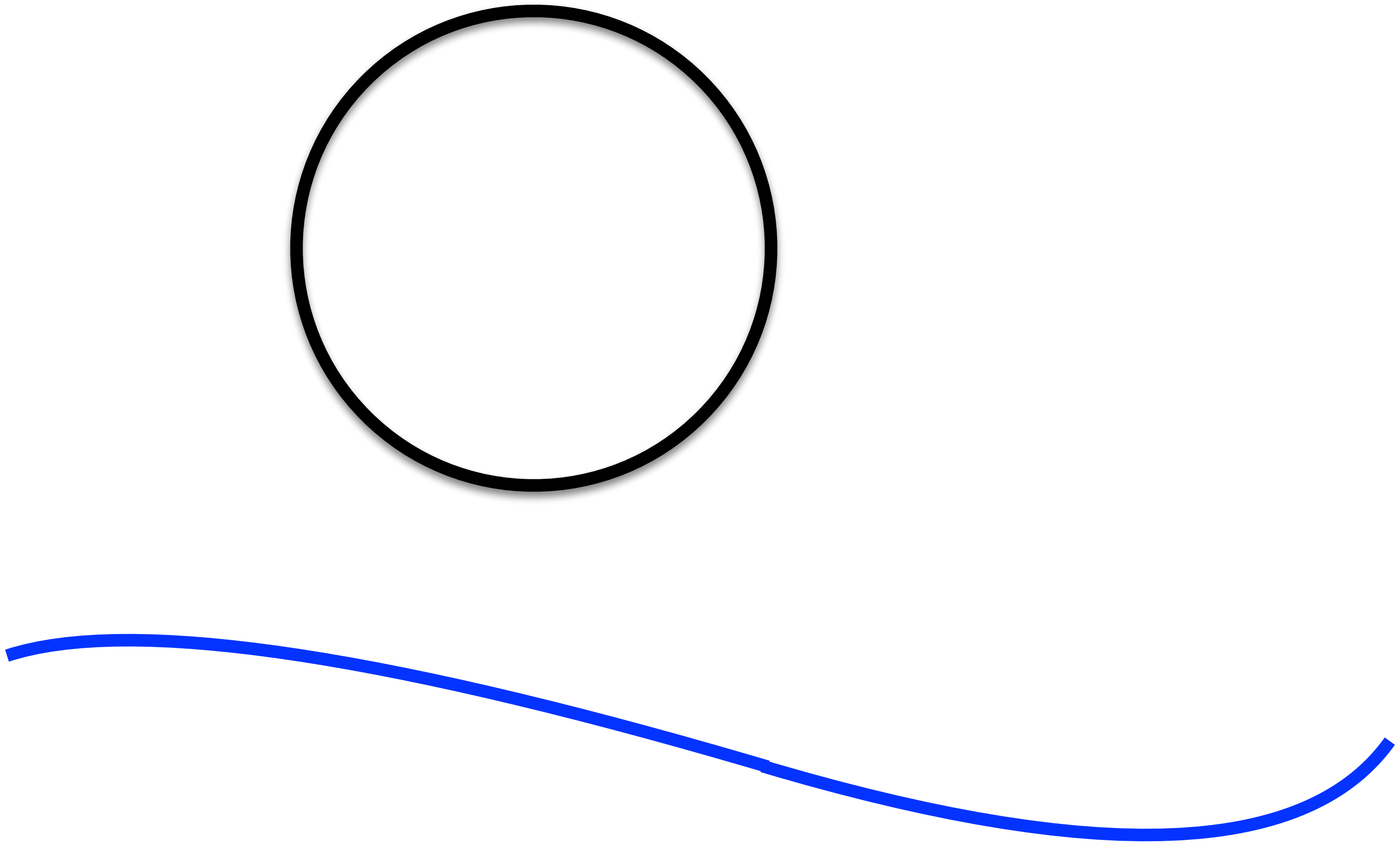
Droplets Walking on a Free Surface



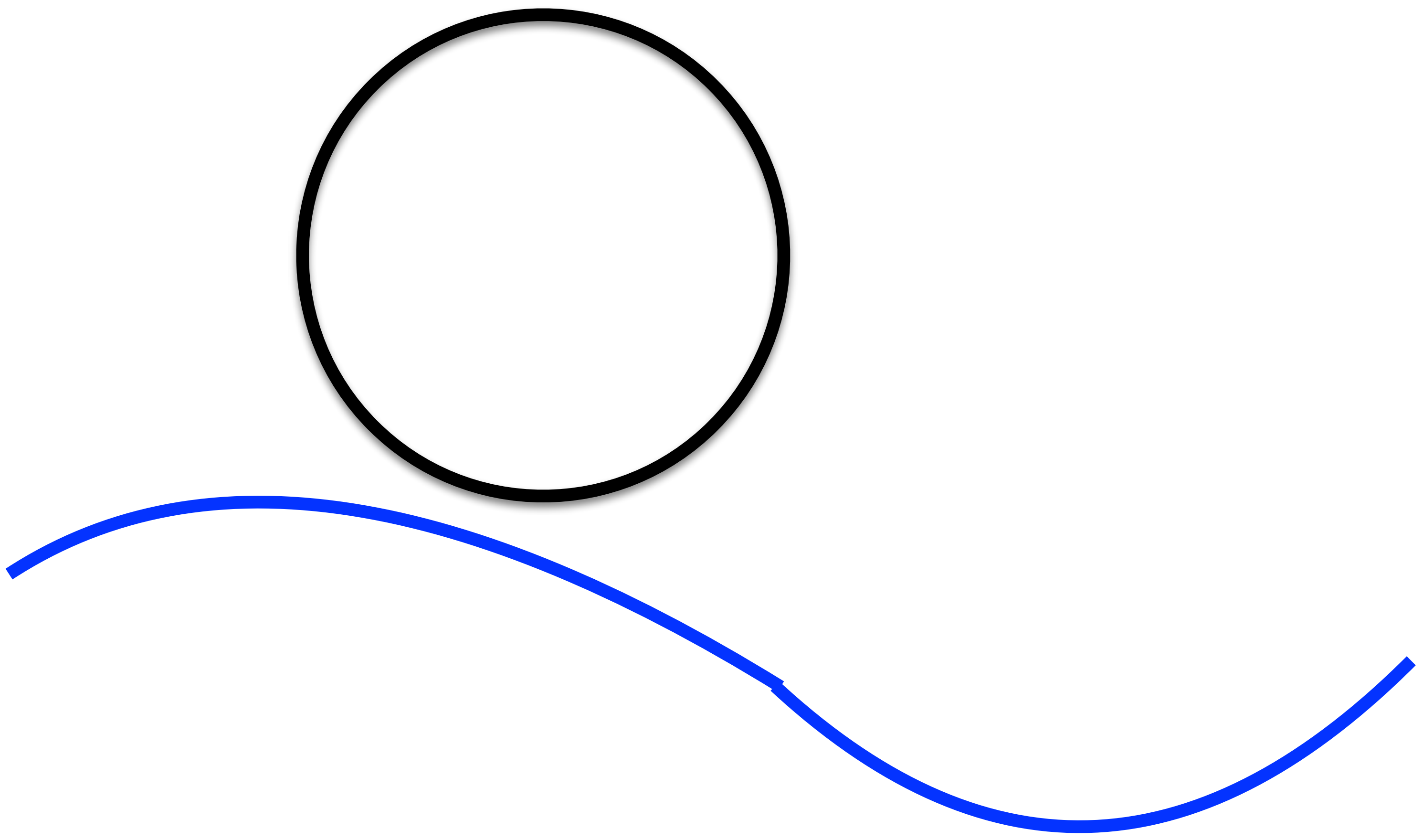
Droplets Walking on a Free Surface



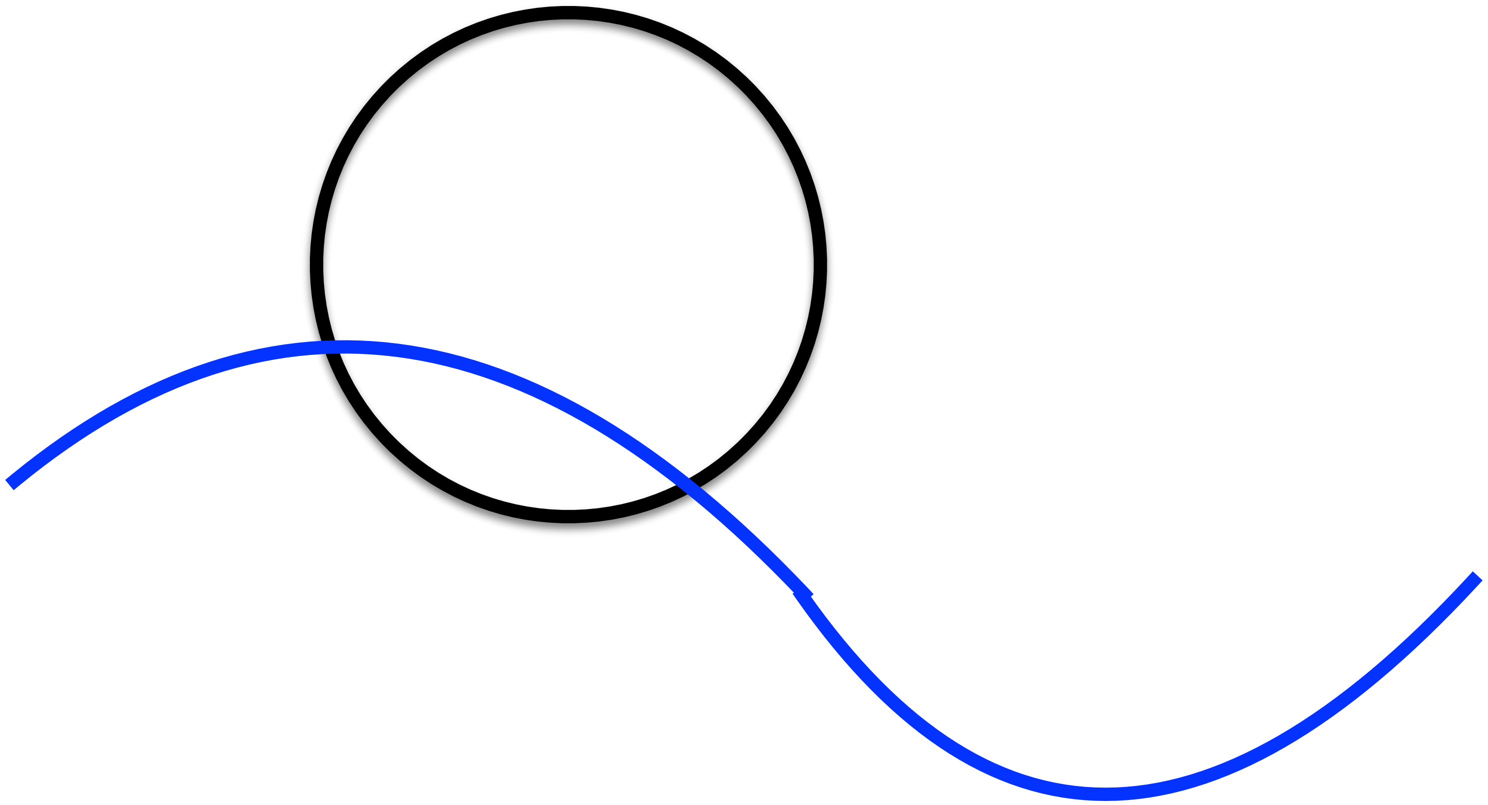
Modelling a Walker



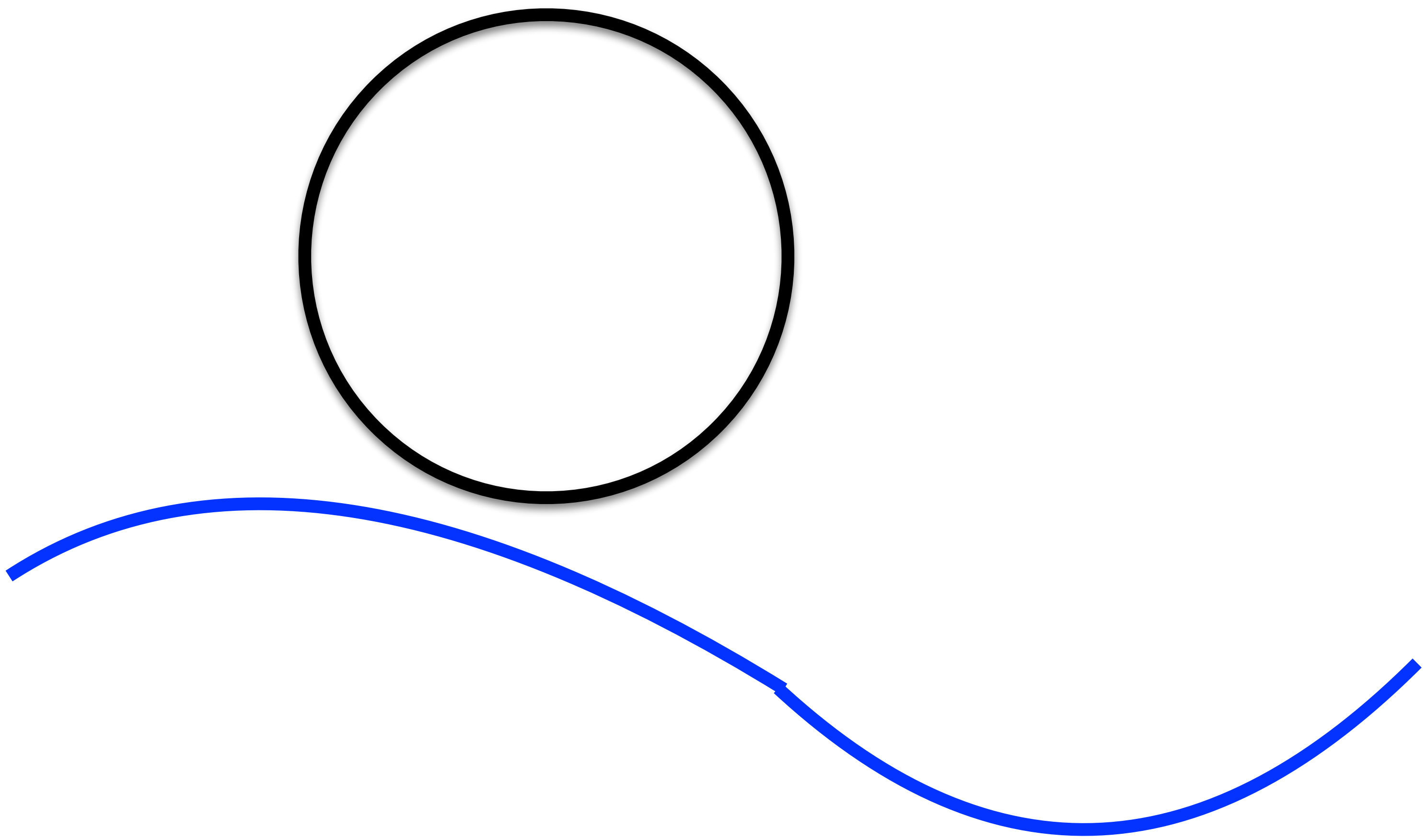
Modelling a Walker



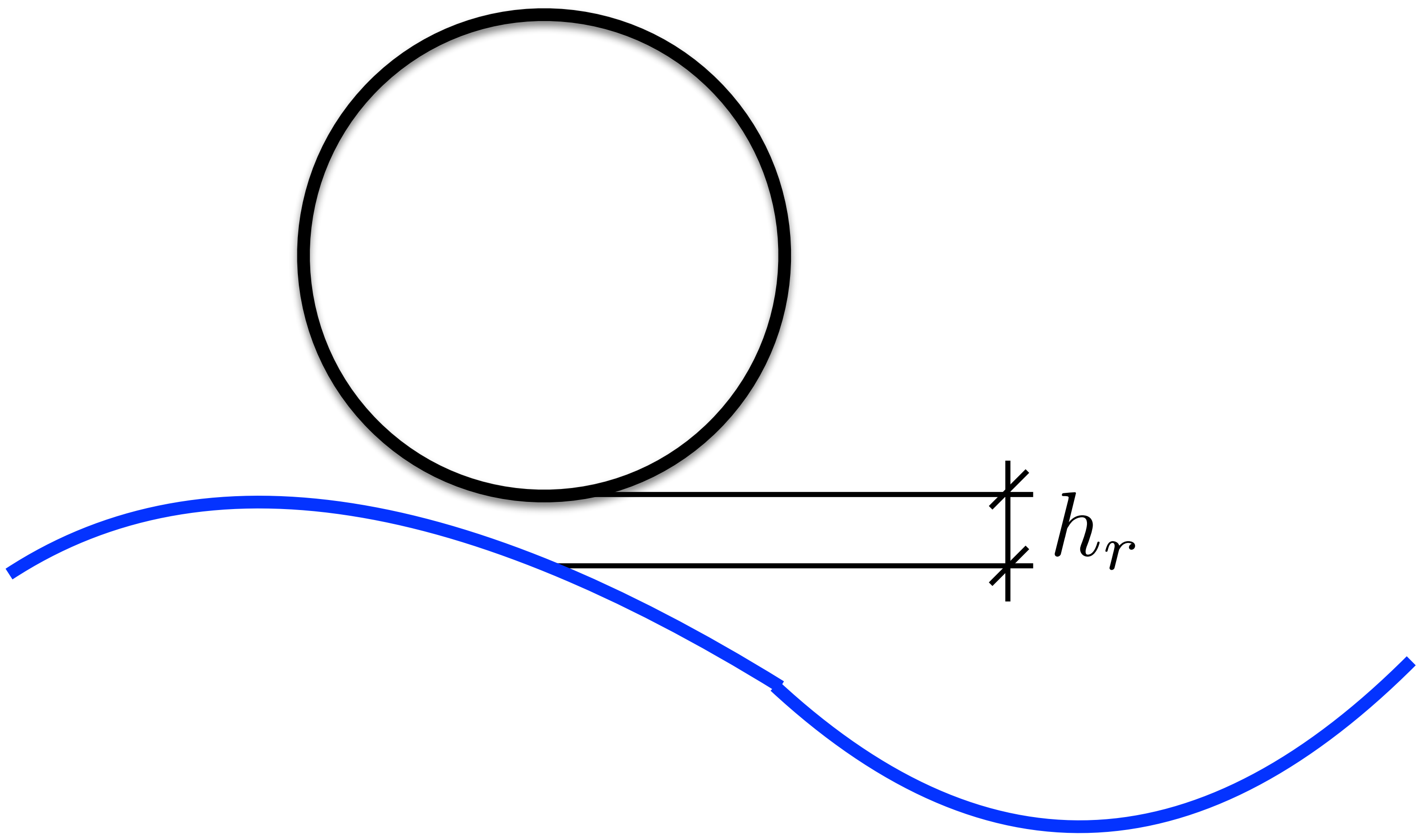
Modelling a Walker



Modelling a Walker



Modelling a Walker



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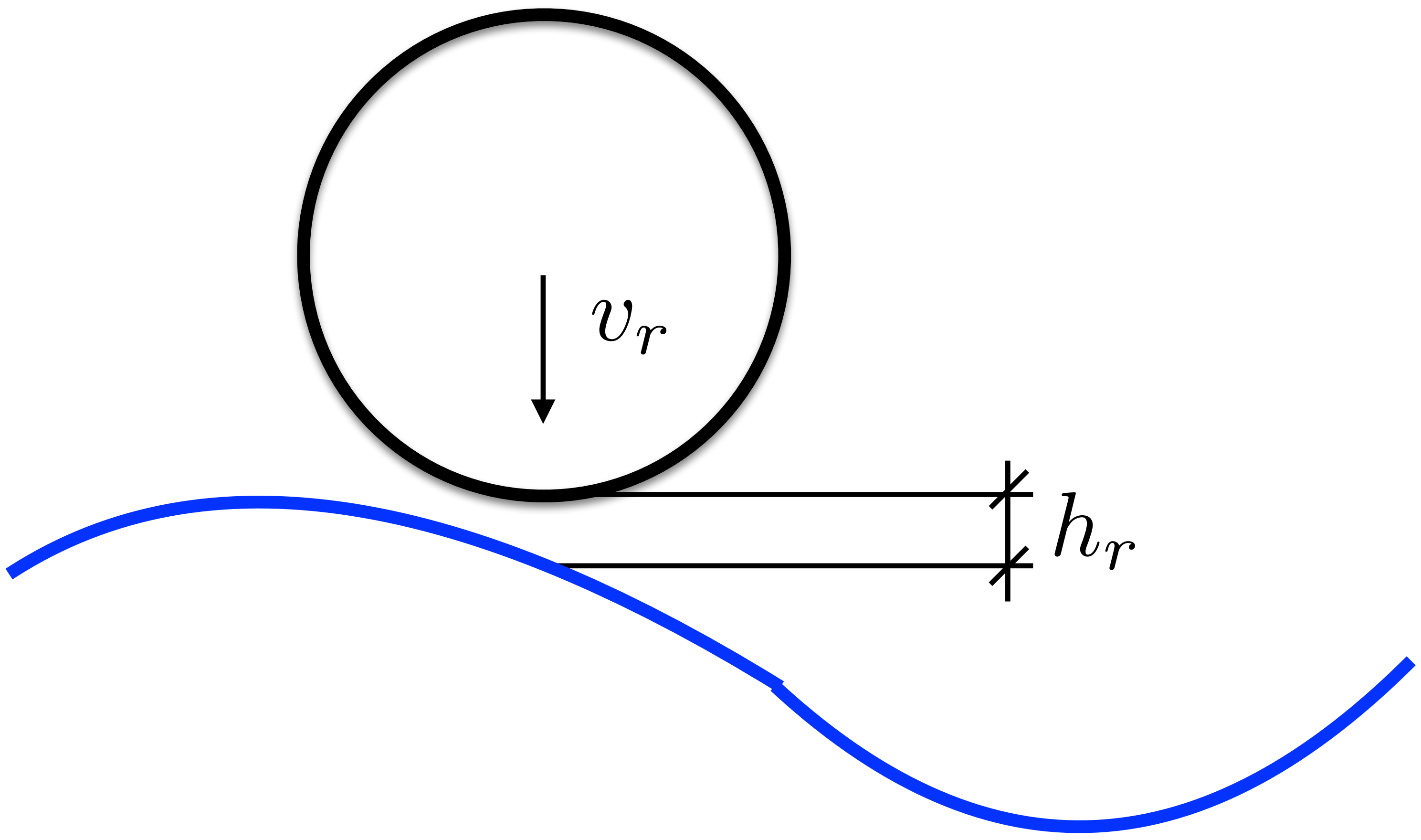
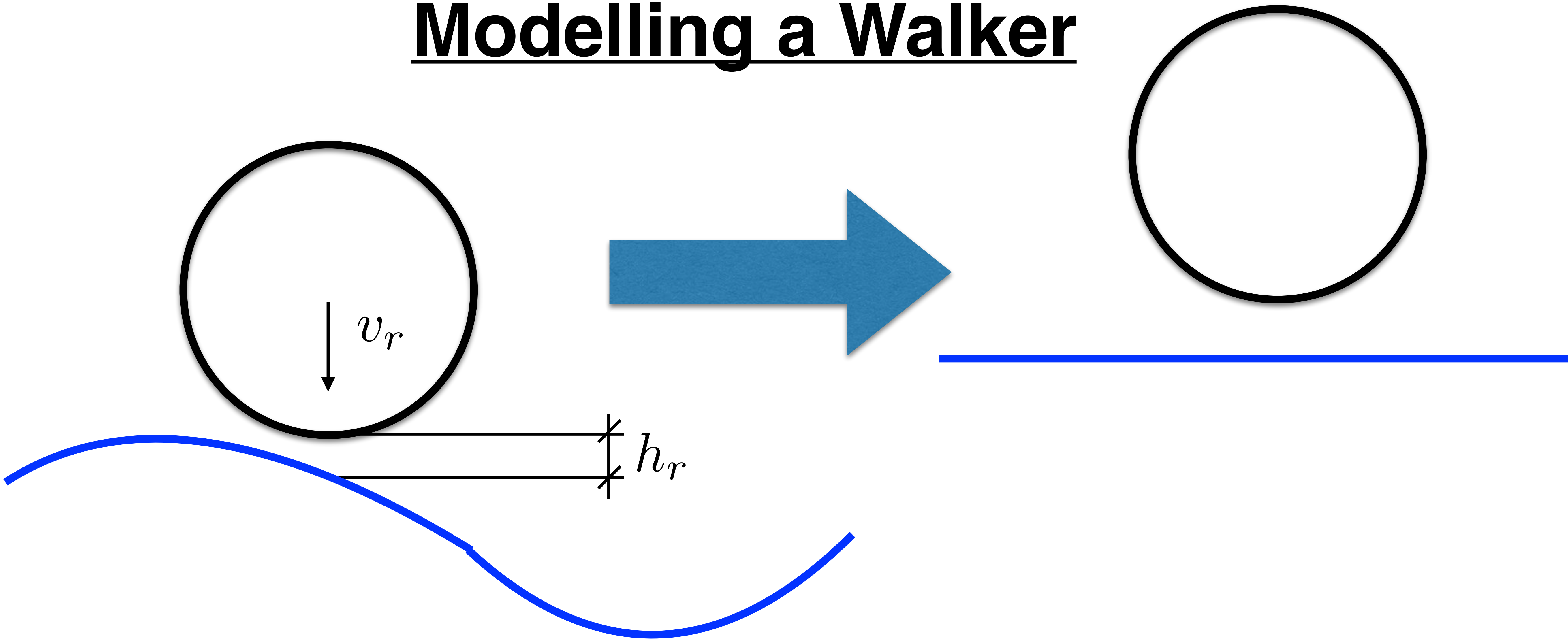
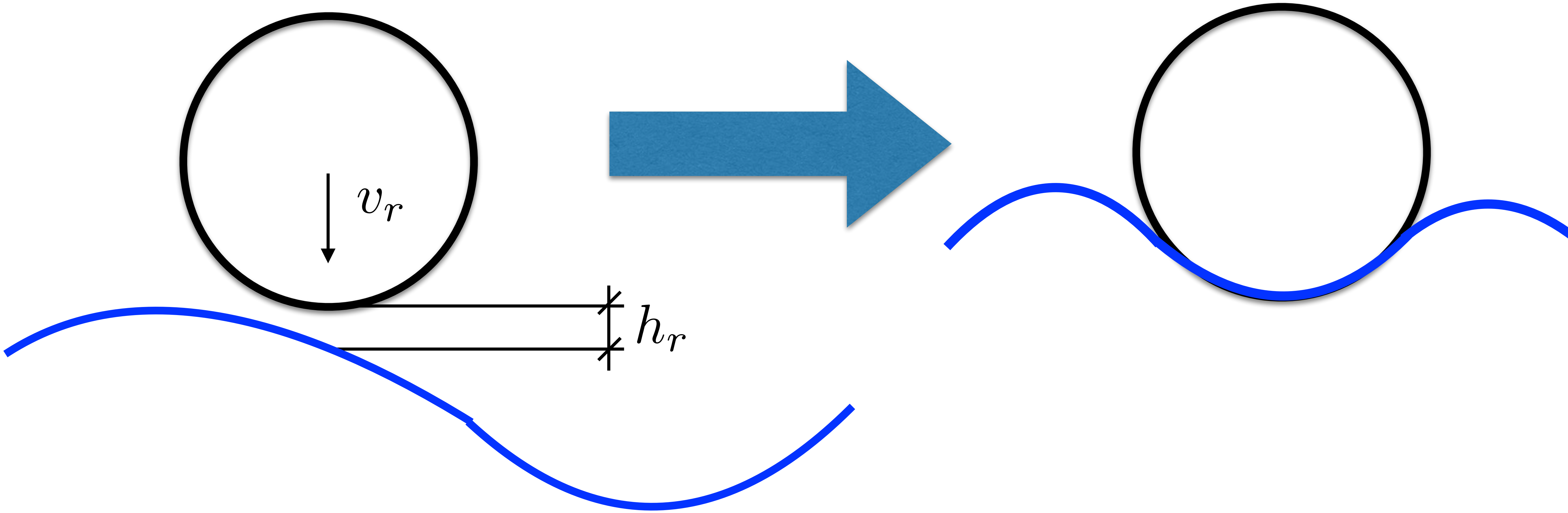


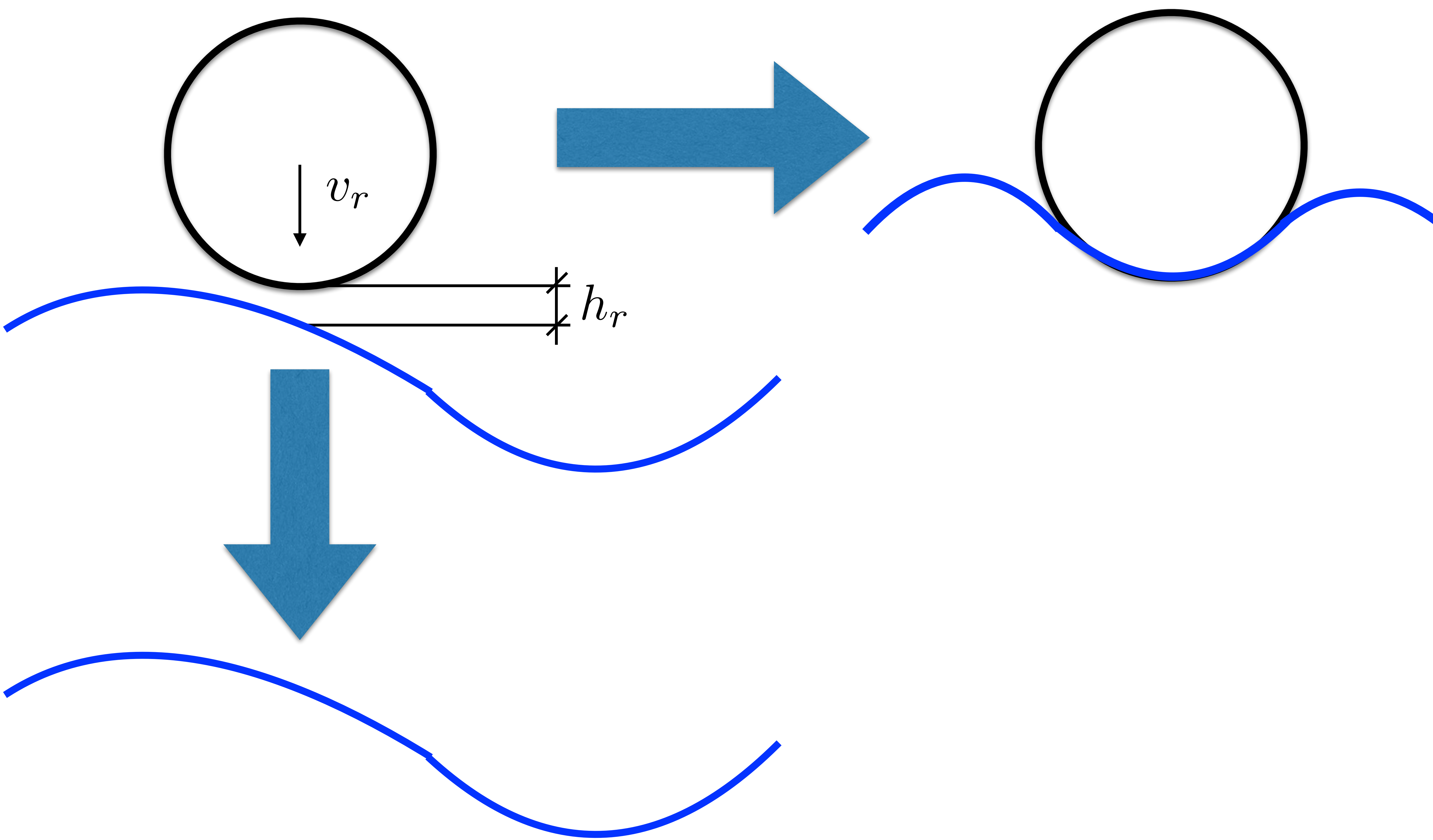
Modelling a Walker



Modelling a Walker



Modelling a Walker



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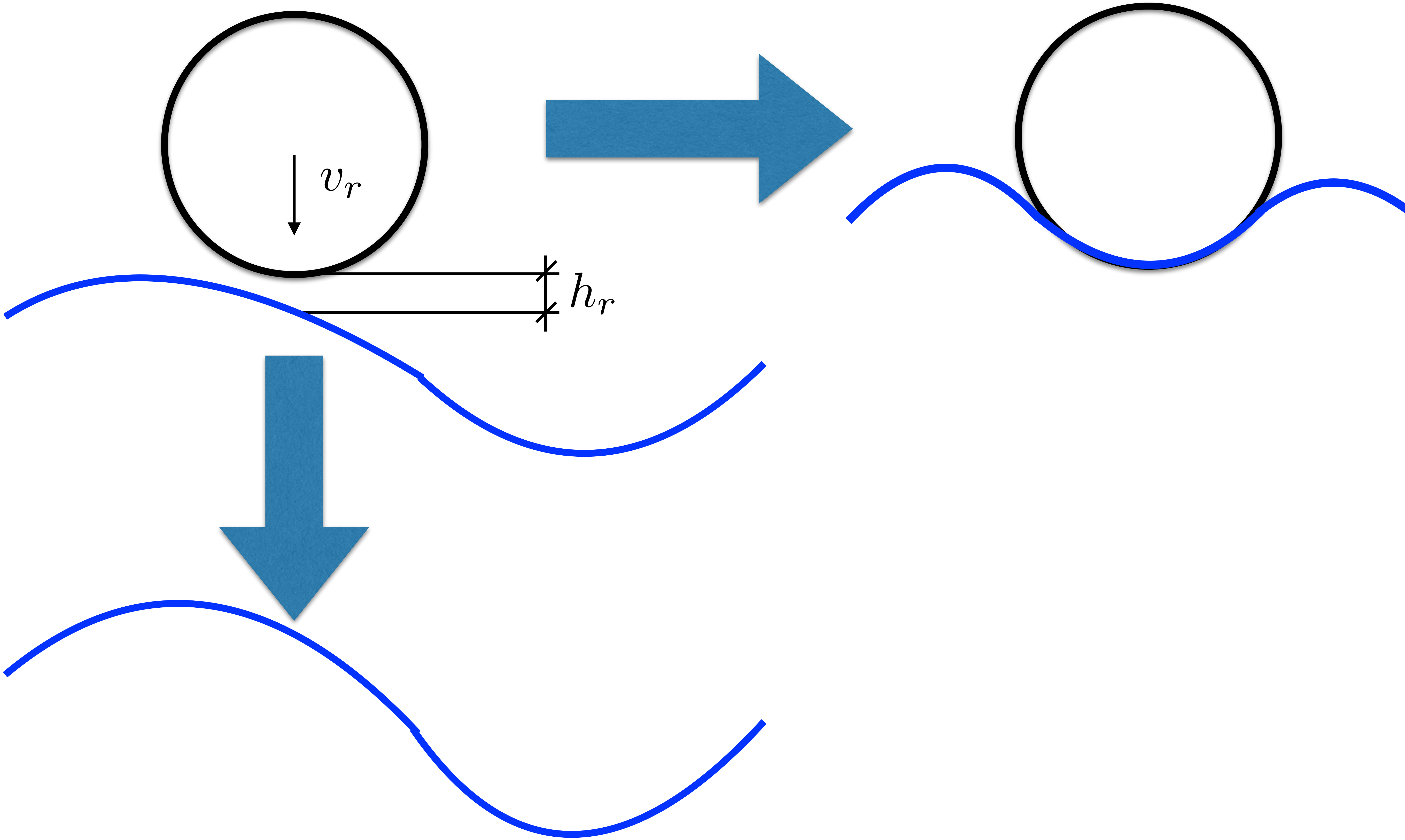
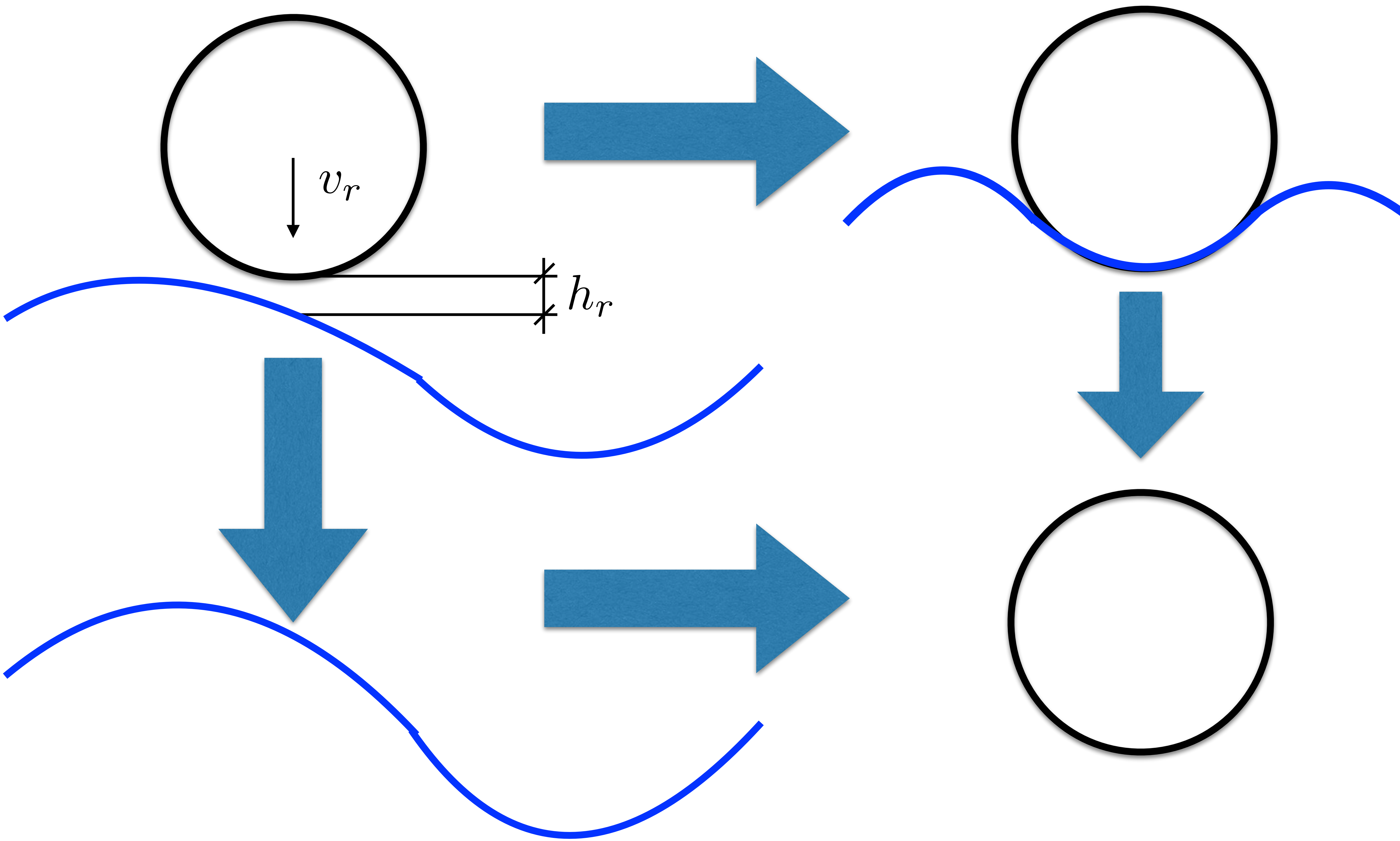
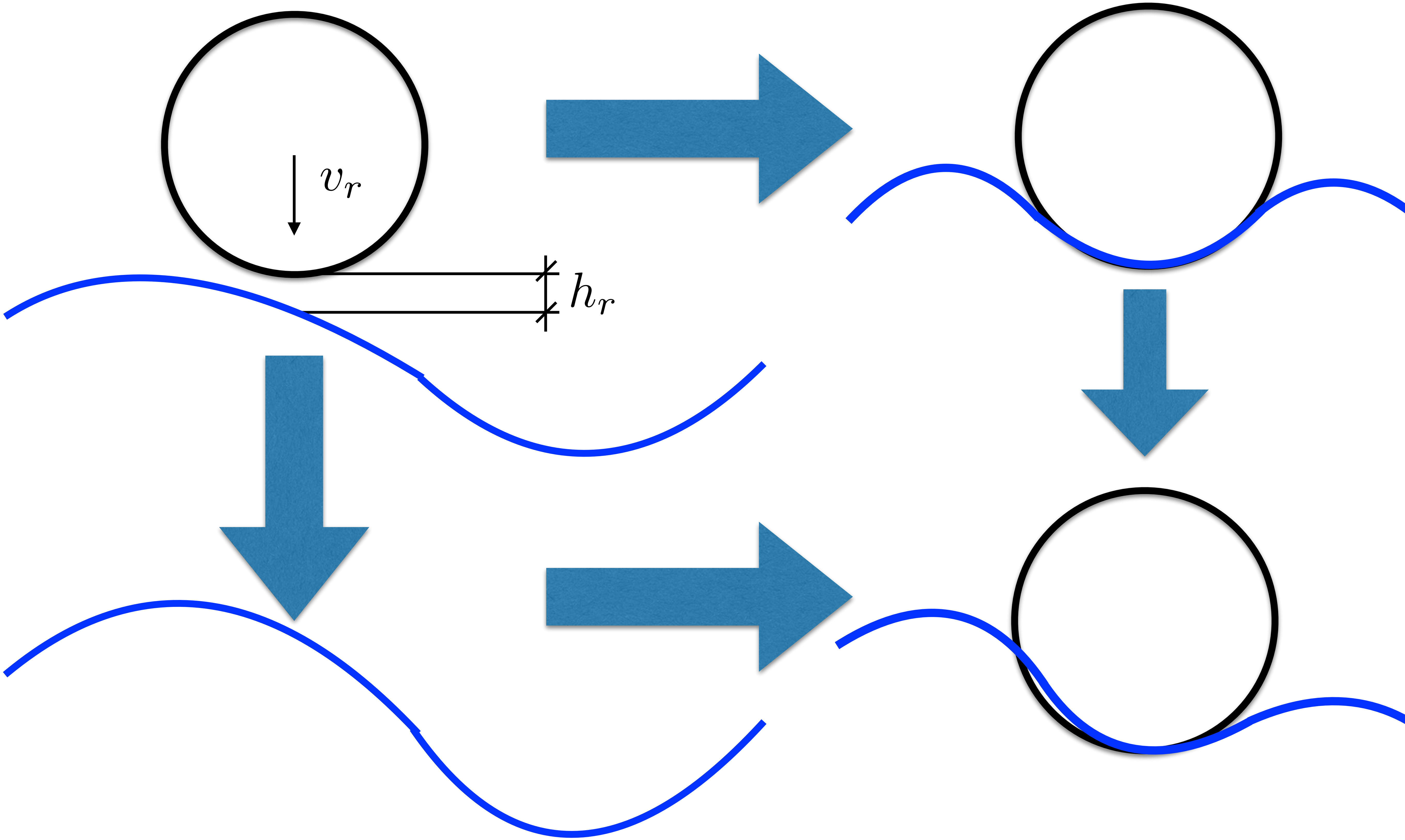
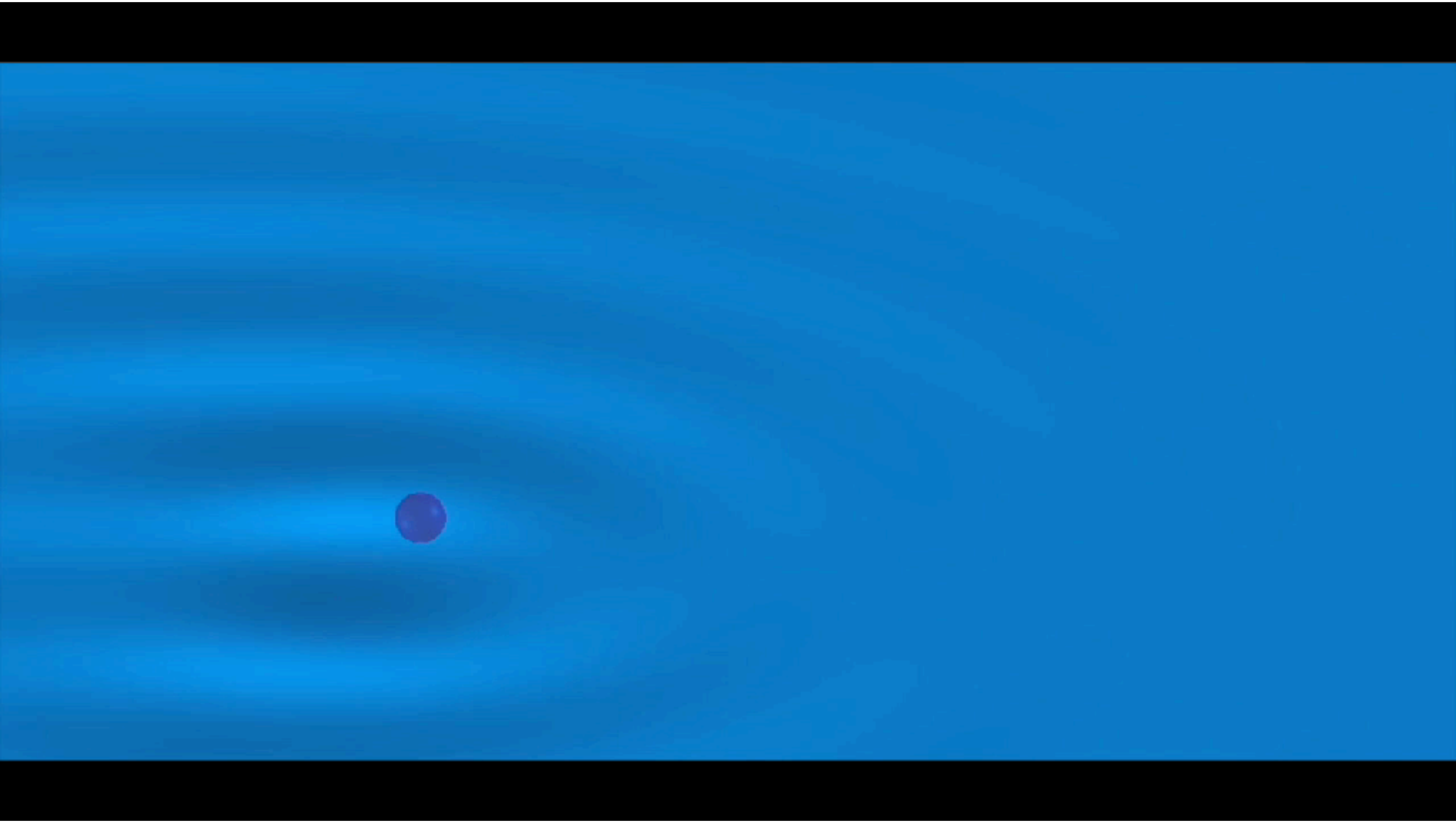


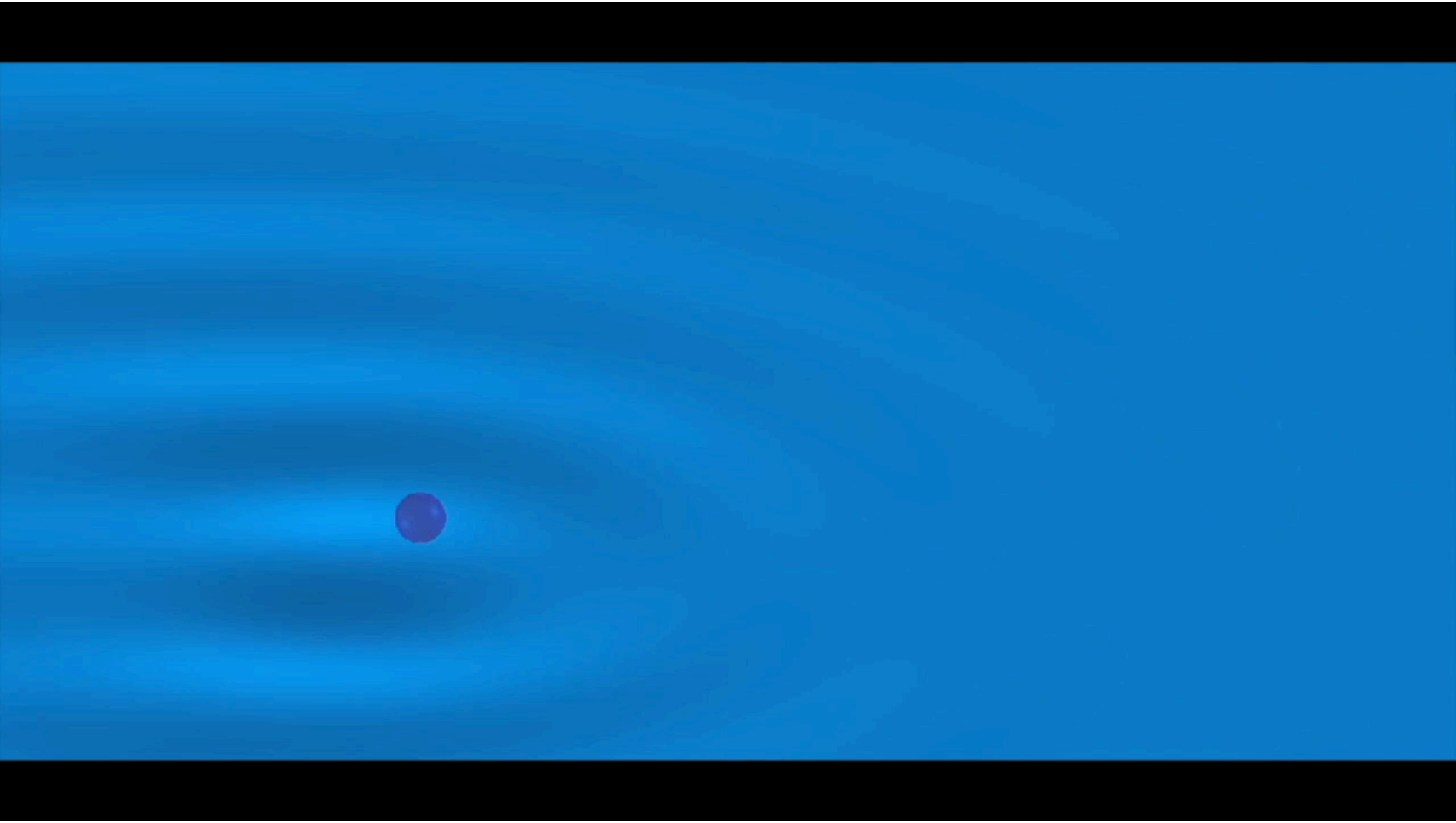
Modelling a Walker



Modelling a Walker







Quasi-normal free-surface impacts, capillary rebounds and application to Faraday walkers

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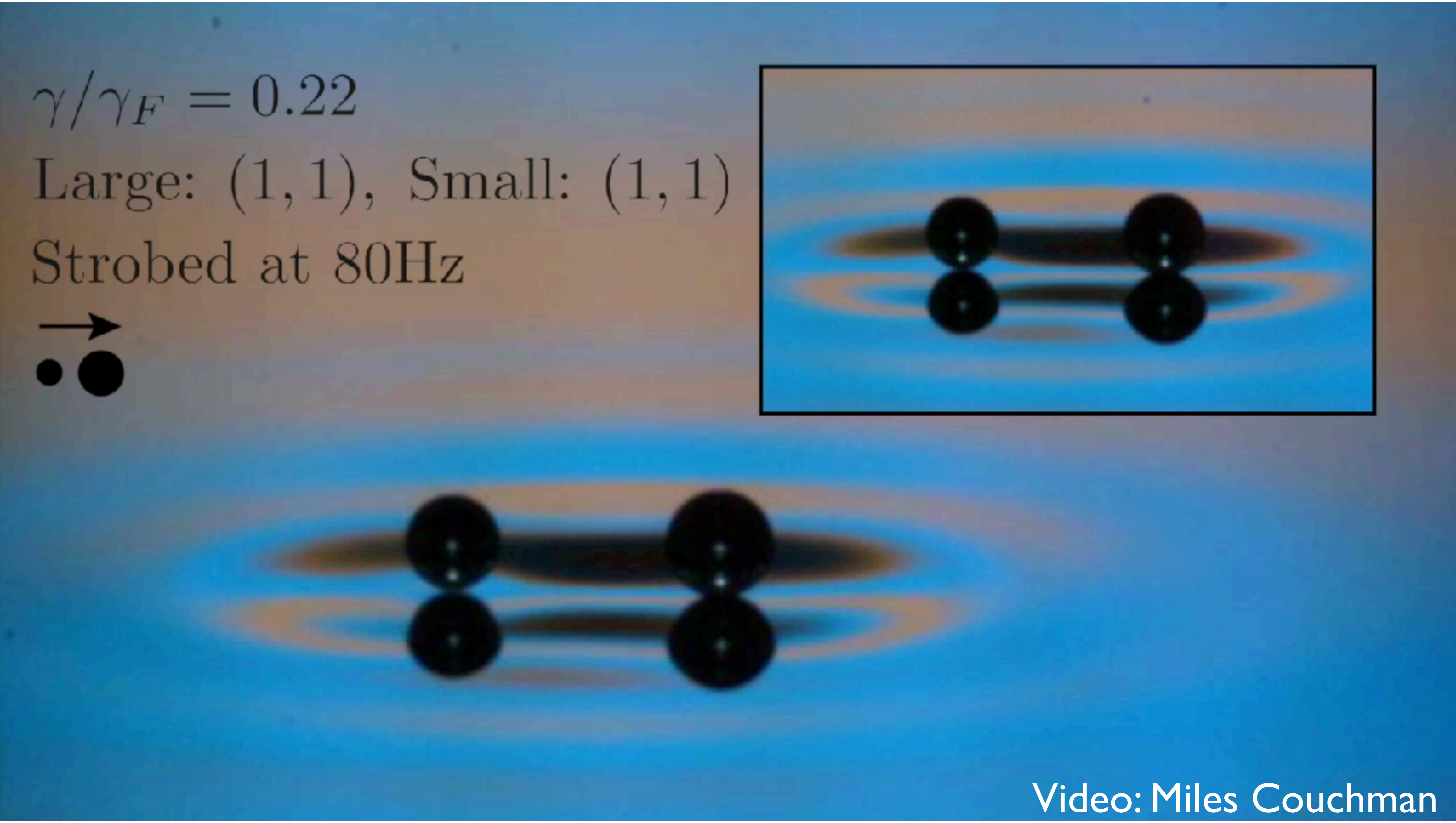
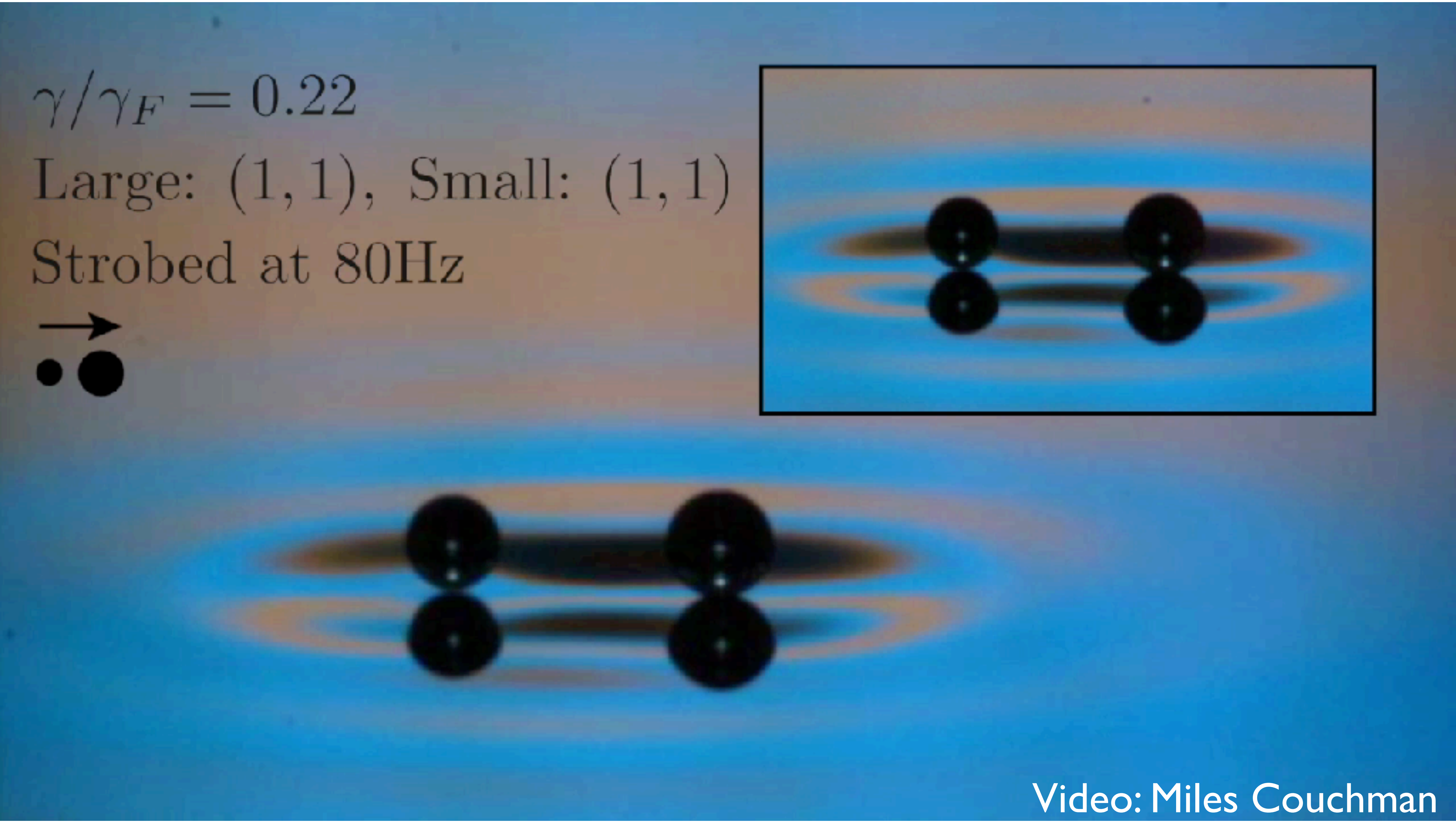
[1]Department of Mathematical Sciences, University of Bath, Bath BA2 7AY, UK

[2]Department of Mathematics, University College London, London WC1E 6BT, UK

(Received 4 December 2018; revised 9 May 2019; accepted 10 May 2019)



Video from Galeano Rios et al 2018 *Ratcheting droplet pairs (Chaos 2018)*



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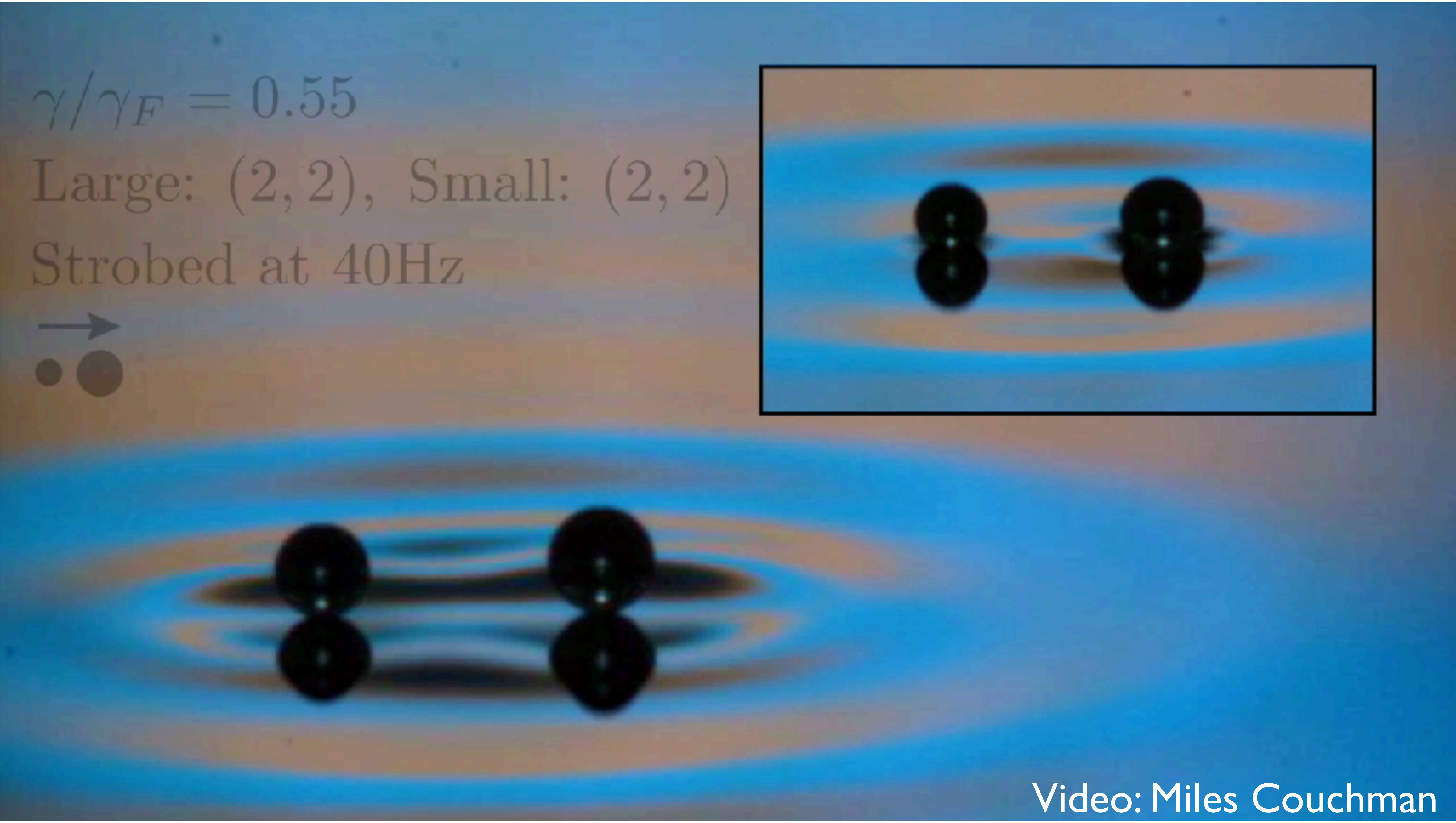
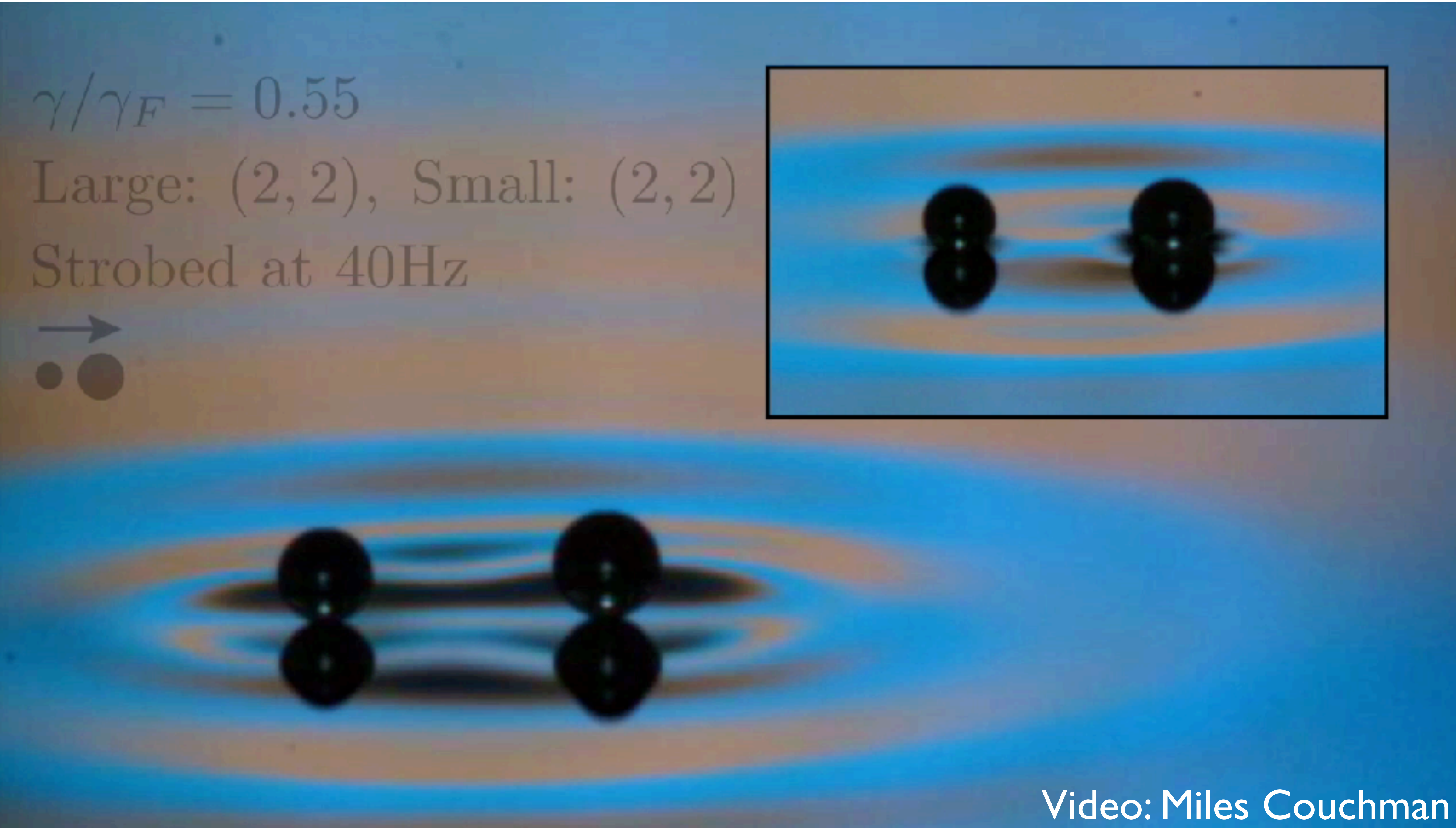


Video from Galeano Rios et al 2018 *Ratcheting droplet pairs (Chaos 2018)*



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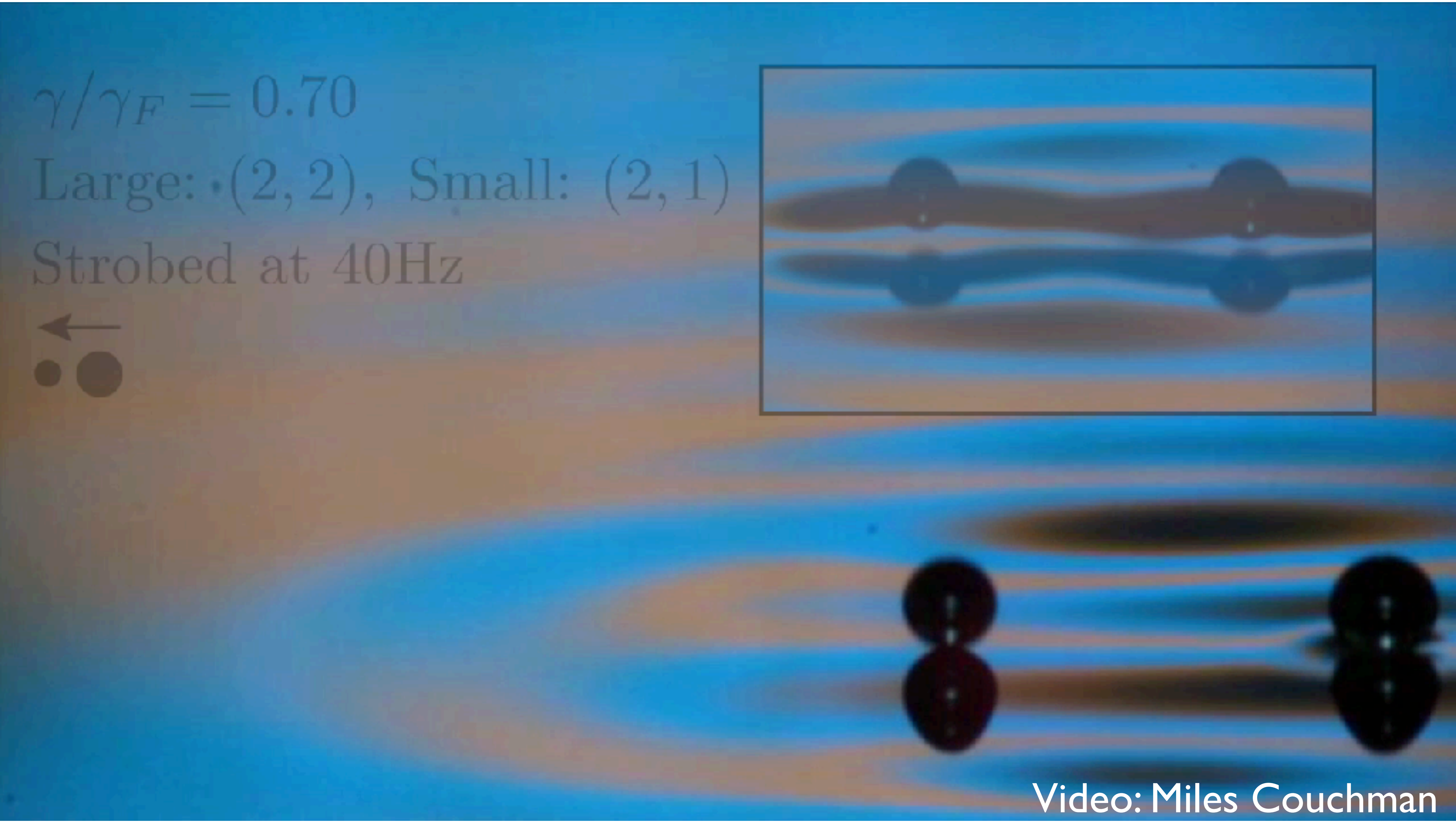

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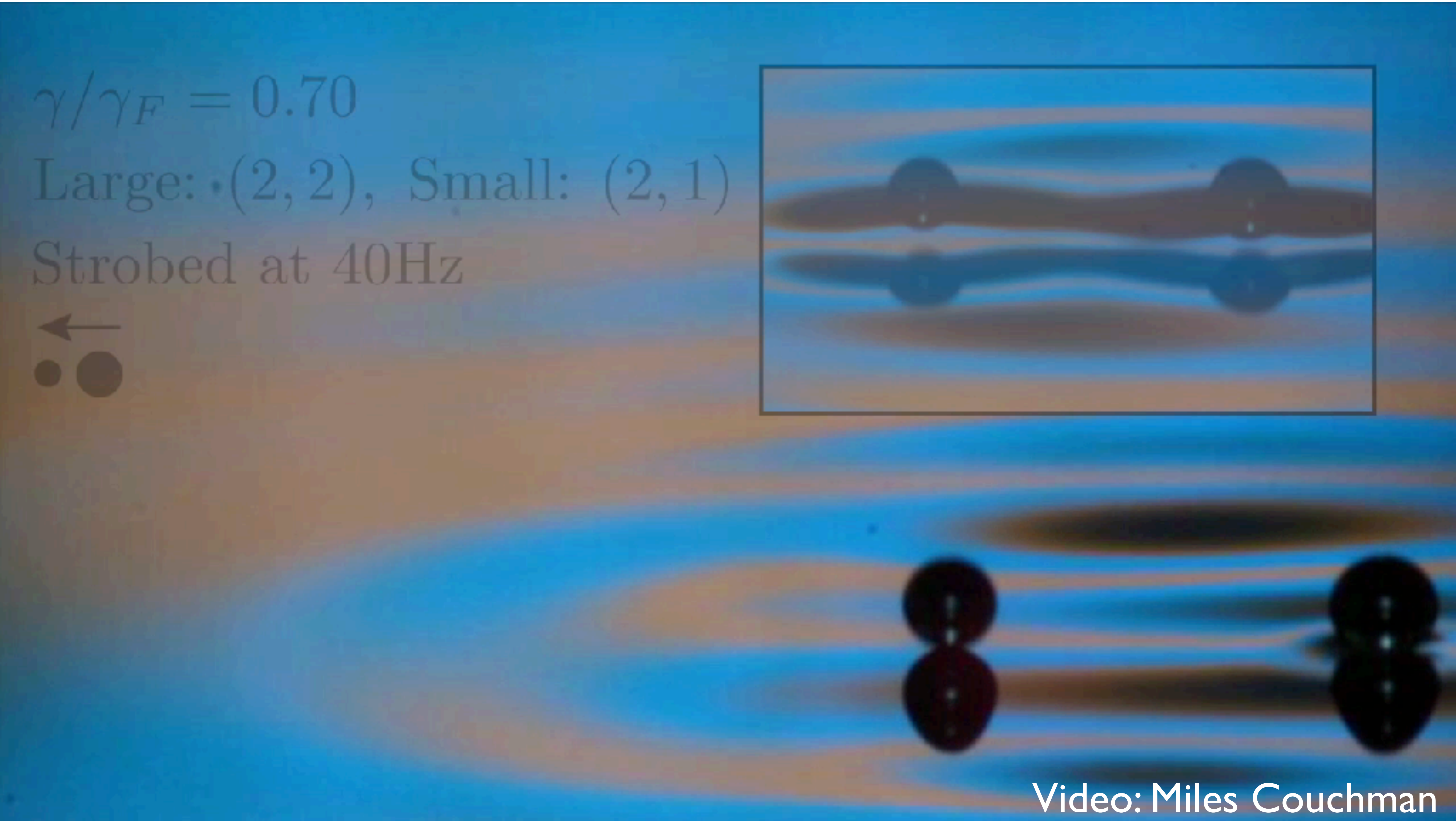


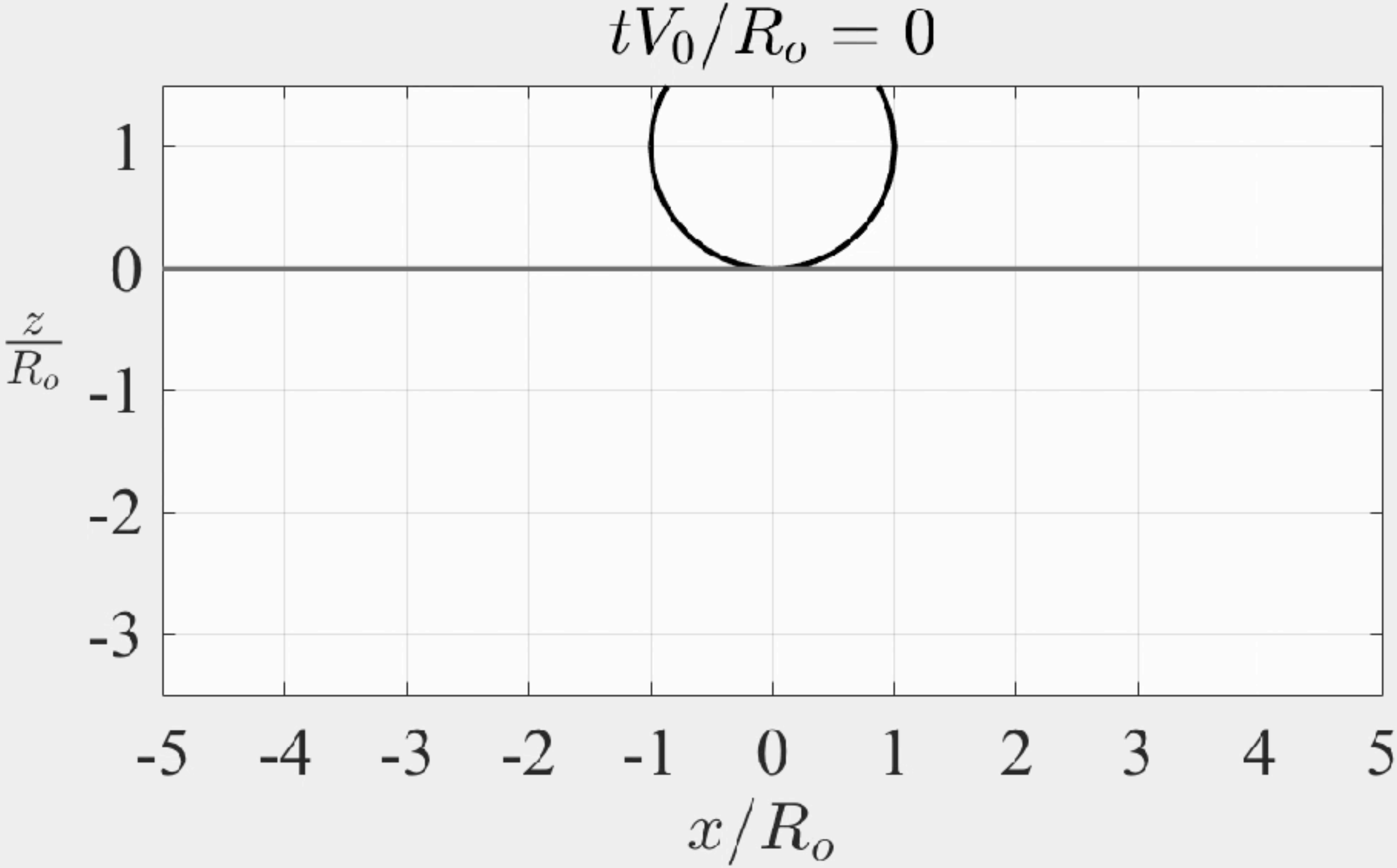
Video from Galeano Rios et al 2018 *Ratcheting droplet pairs (Chaos 2018)*

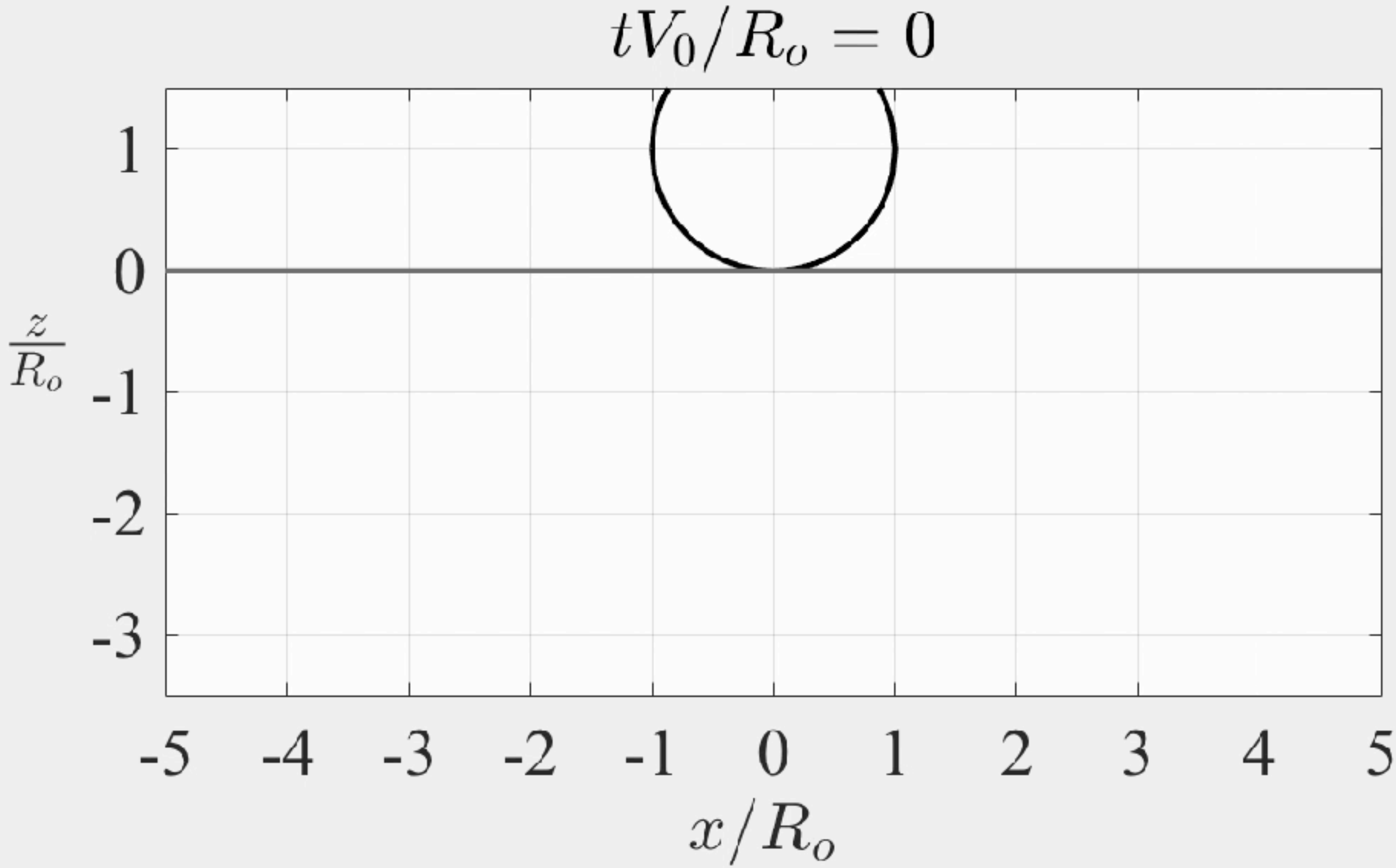
CHAOS **28**, 096112 (2018)

Ratcheting droplet pairs

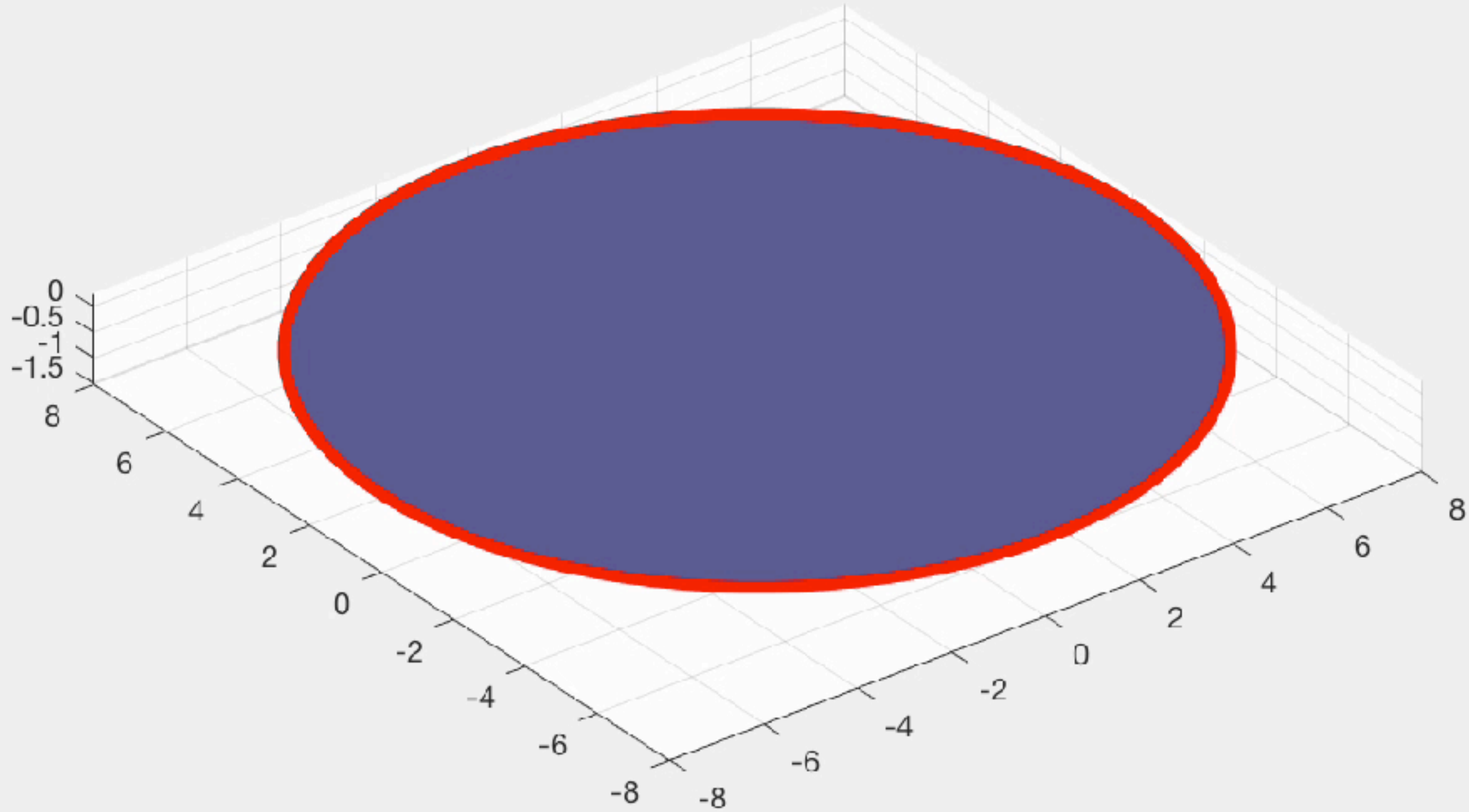
C. A. Galeano-Rios,[1,a)] M. M. P. Couchman,[2,b)] P. Caldairou,[2] and J. W. M. Bush[2,c)]
[1]*Department of Mathematical Sciences, University of Bath, Bath BA2 7AY, United Kingdom*
[2]*Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

(Received 2 April 2018; accepted 31 July 2018; published online 20 September 2018)

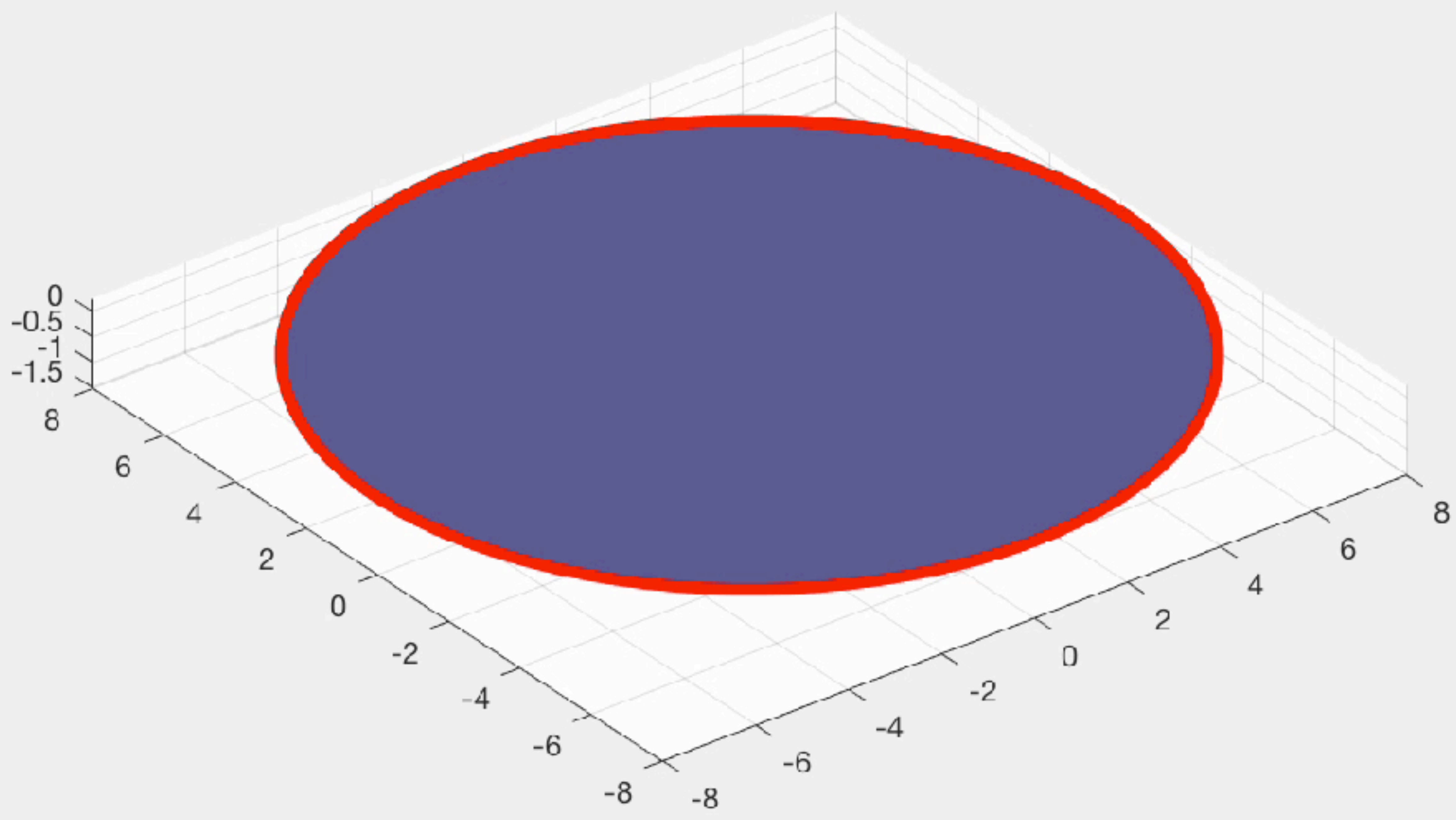

$tV_0/R_o = 0$
1
0
-1
-2
-3
$\frac{z}{R_o}$
-5
-4
-3
-2
-1
0
1
2
3
4
5
x/R_o



$tV_0/R_o = 0$
1
0
-1
-2
-3
$\frac{z}{R_o}$
-5
-4
-3
-2
-1
0
1
2
3
4
5
x/R_o

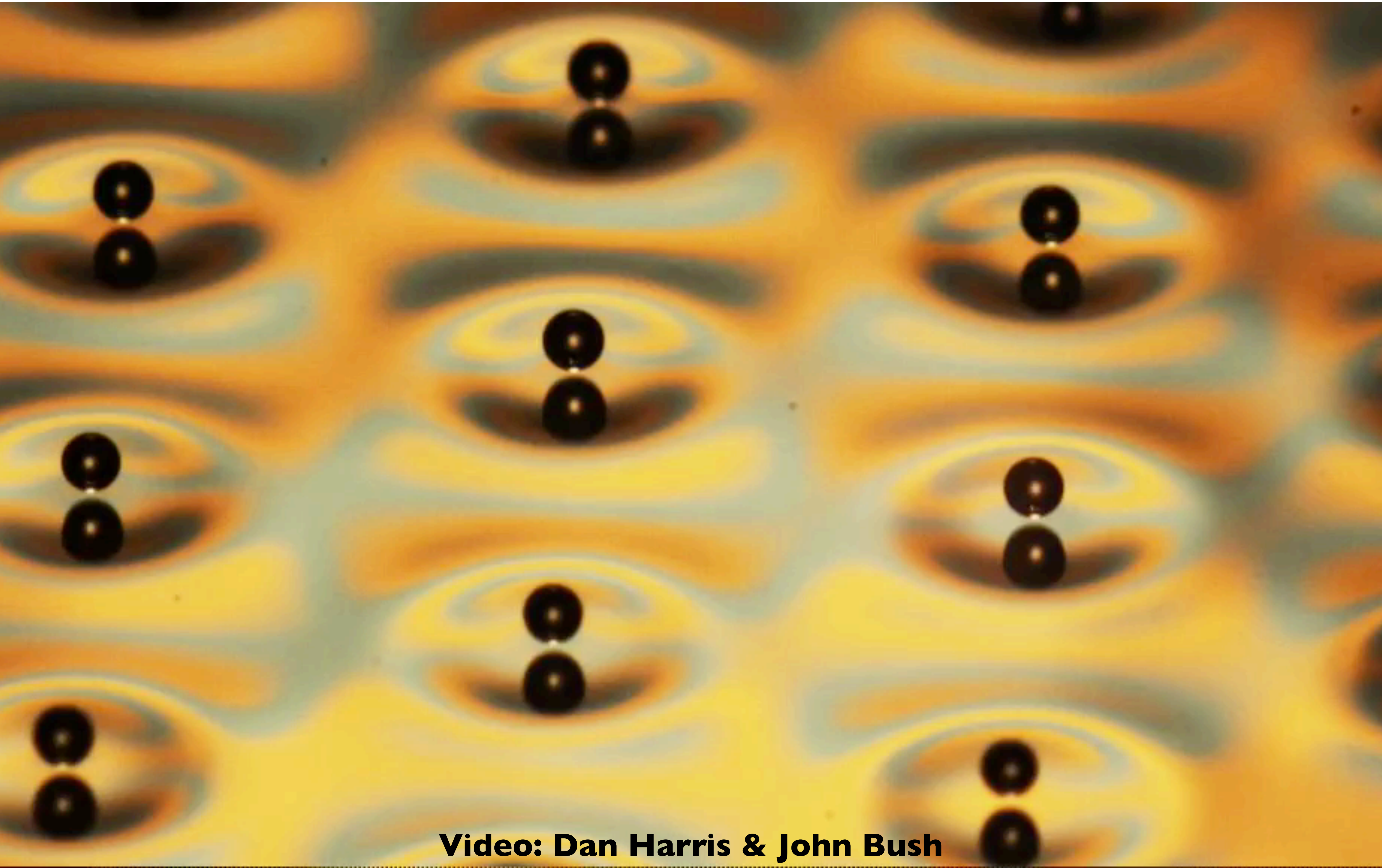


0
-0.5
-1
-1.5
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8



0
-0.5
-1
-1.5
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8

Thanks for your attention



Thanks for your attention