

# Bouncing and floating on a free surface: The kinematic match 

## Carlos Galeano-Rios

## HyWEC 2

June 20th, 2019
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## EPSRC

Engineering and Physical Sciences
Research Council

## Experiments by Daniel Harris, Brown University

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$$
t=t_{0}+\delta t
$$



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## Falling Sphere = Moving Ceiling



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Perfectly Hydrophobic Rigid Sphere

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Perfectly Hydrophobic Rigid Sphere

in the contact area

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h(t)+z(x, y)=\eta(x, y, t)
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Perfectly Hydrophobic Rigid Sphere
in the contact area $\longrightarrow h(t)+z(x, y)=\eta(x, y, t)$

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h_{t t}=-\frac{1}{F r}-D h_{t}+\frac{1}{M} \int_{r \leqslant r_{c}} p_{s} \mathrm{~d} A,
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## 0 fitting parameters

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A Non-local Formulation in Physical Space

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\begin{aligned}
\Delta \phi & =0, & & z \leqslant 0, \\
\eta_{t} & =\frac{2}{R e} \Delta_{H} \eta+\phi_{z}, & & z=0, \\
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$$
R e=V_{0} R_{o} / \nu
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F r=V_{0}^{2} /(g R o)
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\eta \rightarrow 0 & \text { when } & \sqrt{x^{2}+y^{2}} \rightarrow \infty \\
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\begin{aligned}
& \left.h_{t t}=-\frac{1}{F r} h-D h_{t}+\frac{1}{M} \int_{r \leq r} P_{s}\right)^{A} \\
& \eta=h+z_{s}, \quad \text { where } r \leq r_{c} ;
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$\eta<h+z_{s}$, where $r_{c}<r<R_{o}$;

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p_{s}=0, \quad \text { where } \quad r>r_{c} ;
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# Numerical Implementation 

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Q W^{j+1}=F^{j},
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\boldsymbol{Q}=\left[\begin{array}{ccccc}
\left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & -\delta t N & 0 & 0 & 0 \\
\delta t\left(\frac{1}{F r} \boldsymbol{I}-\frac{1}{W e} \Delta_{H}\right) & \left(I-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta t \boldsymbol{I} & 0 & 0 \\
0 & 0 & -\delta t \frac{A}{M} & (1+\delta t D) & 0  \tag{0}\\
0 & 0 & 0 & -\delta t & 1
\end{array}\right]
$$

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0 & 0 & -\delta t \frac{A}{M} & (1+\delta t D) & 0 \\
0 & 0 & 0 & -\delta t & 1
\end{array}\right], \\
W^{j+1}=\left[\begin{array}{lllll}
\eta^{j+1} & \phi^{j+1} & p_{s}^{j+1} & h_{t}^{j+1} & h^{j+1}
\end{array}\right]^{\mathrm{T}},
\end{gathered}
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\boldsymbol{Q}=\left[\begin{array}{cccc}
\left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & -\delta t N & 0 & 0 \\
\delta t\left(\frac{1}{F r} \boldsymbol{I}-\frac{1}{W e} \Delta_{H}\right) & \left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta t \boldsymbol{I} & 0 \\
0 & 0 & -\delta t \frac{A}{M} & (1+\delta t D) \\
0 & 0 & 0 & -\delta t
\end{array}\right. \\
W^{j+1}=\left[\begin{array}{llll}
\eta^{j+1} & \phi^{j+1} & p_{s}^{j+1} & h_{t}^{j+1} \\
h^{j+1}
\end{array}\right]^{\mathrm{T}}, \\
F^{j}=\left[\begin{array}{lll}
\eta^{j} & \left(\phi^{j}+\frac{1}{W e}\left(\kappa-\Delta_{H}\right) \eta^{j+1}\right) & \left(h_{t}^{j}-\delta t \frac{1}{F r}\right)
\end{array} h^{j}\right.
\end{array}\right]^{\mathrm{T}} .
$$

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{W_{k}^{j+1}}=F_{k}^{j},
$$

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j},
$$

$$
\boldsymbol{Q}_{k}=\left[\begin{array}{ccc}
\left(\boldsymbol{I}^{k^{\prime}}-\frac{2 \delta t}{R e} \Delta_{H}^{k^{\prime}}\right) & -\delta t N & 0  \tag{0}\\
\delta t\left(\frac{1}{F r} \boldsymbol{l}^{\mu^{\prime}}-\frac{1}{W e} \Delta_{H}^{k^{\prime}}\right) & \left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta \boldsymbol{\boldsymbol { l } ^ { k }}
\end{array}\right.
$$

$$
\left.\begin{array}{c}
a_{k} \\
b_{k}
\end{array}\right]
$$

$$
\left.\begin{array}{llccc}
0 & 0 & -\delta t \frac{A^{k}}{M} & (1+\delta t D) & 0 \\
0 & 0 & 0 & -\delta t & 1
\end{array}\right]
$$

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j},
$$

$$
\boldsymbol{Q}_{k}=\left[\begin{array}{ccccc}
\left(\boldsymbol{l}^{k^{\prime}}-\frac{2 \delta t}{R e} \Delta_{H}^{k^{\prime}}\right) & -\delta t N & 0 & 0 & a_{k} \\
\delta t\left(\frac{1}{F r} \boldsymbol{l}^{k^{\prime}}-\frac{1}{W e} \Delta_{H}^{k^{\prime}}\right) & \left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta t \boldsymbol{l}^{k} & 0 & b_{k}  \tag{0}\\
0 & 0 & -\delta t \frac{A^{k}}{M} & (1+\delta t D) & 0 \\
0 & 0 & 0 & -\delta t & 1
\end{array}\right]
$$

$W^{j+1}=\left[\begin{array}{lllll}\eta^{j+1, k^{\prime}} & \phi^{j+1} & p_{s}^{j+1, k} & h_{t}^{j+1} & h^{j+1}\end{array}\right]^{\mathrm{T}}$,

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j},
$$

$$
\boldsymbol{Q}_{k}=\left[\begin{array}{ccccc}
\left(\boldsymbol{l}^{\prime}-\frac{2 \delta t}{R e} \Delta_{H}^{k^{\prime}}\right) & -\delta t N & 0 & 0 & a_{k} \\
\delta t\left(\frac{1}{F r} \boldsymbol{\mu}^{\prime}-\frac{1}{W e} \Delta_{H}^{k^{\prime}}\right) & \left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta \boldsymbol{\boldsymbol { l } ^ { k }} & 0 & b_{k} \\
0 & 0 & -\delta t \frac{A^{k}}{M} & (1+\delta t D) & 0 \\
0 & 0 & 0 & -\delta t & 1
\end{array}\right]
$$

$$
W^{j+1}=\left[\begin{array}{lllll}
\eta^{j+1, k^{\prime}} & \phi^{j+1} & p_{s}^{j+1, k} & h_{t}^{j+1} & h^{j+1}
\end{array}\right]^{\mathrm{T}},
$$

Superindex $k$

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j},
$$

$$
\boldsymbol{Q}_{k}=\left[\begin{array}{ccccc}
\left(\boldsymbol{l}^{k^{\prime}}-\frac{2 \delta t}{R e} \Delta_{H}^{k^{\prime}}\right) & -\delta t N & 0 & 0 & a_{k} \\
\delta t\left(\frac{1}{F r} \boldsymbol{l}^{k^{\prime}}-\frac{1}{W e} \Delta_{H}^{k^{\prime}}\right) & \left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta t \boldsymbol{l}^{k} & 0 & b_{k}  \tag{0}\\
0 & 0 & -\delta t \frac{A^{k}}{M} & (1+\delta t D) & 0 \\
0 & 0 & 0 & -\delta t & 1
\end{array}\right]
$$

$$
W^{j+1}=\left[\begin{array}{lllll}
\eta^{j+1, k^{\prime}} & \phi^{j+1} & p_{s}^{j+1, k} & h_{t}^{j+1} & h^{j+1}
\end{array}\right]^{\mathrm{T}}
$$

Superindex $k \longmapsto$ we kept the first $k$ columns

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j},
$$

$$
\boldsymbol{Q}_{k}=\left[\begin{array}{ccccc}
\left(\boldsymbol{l}^{k^{\prime}}-\frac{2 \delta t}{R e} \Delta_{H}^{k^{\prime}}\right) & -\delta t N & 0 & 0 & a_{k} \\
\delta t\left(\frac{1}{F r} \boldsymbol{l}^{k^{\prime}}-\frac{1}{W e} \Delta_{H}^{k^{\prime}}\right) & \left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta t \boldsymbol{I}^{k} & 0 & b_{k}  \tag{0}\\
0 & 0 & -\delta t \frac{A^{k}}{M} & (1+\delta t D) & 0 \\
0 & 0 & 0 & -\delta t & 1
\end{array}\right]
$$

$$
W^{j+1}=\left[\begin{array}{lllll}
\eta^{j+1, k^{\prime}} & \phi^{j+1} & p_{s}^{j+1, k} & h_{t}^{j+1} & h^{j+1}
\end{array}\right]^{\mathrm{T}}
$$

Superindex $k \longmapsto$ we kept the first $k$ columns Superindex $k^{\prime}$

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j},
$$

$$
\boldsymbol{Q}_{k}=\left[\begin{array}{ccccc}
\left(\boldsymbol{I}^{k^{\prime}}-\frac{2 \delta t}{R e} \Delta_{H}^{k^{\prime}}\right) & -\delta t N & 0 & 0 & a_{k} \\
\delta t\left(\frac{1}{F r} \boldsymbol{l}^{k^{\prime}}-\frac{1}{W e} \Delta_{H}^{k^{\prime}}\right) & \left(\boldsymbol{I}-\frac{2 \delta t}{R e} \Delta_{H}\right) & \delta t \boldsymbol{l}^{k} & 0 & b_{k} \\
0 & 0 & -\delta t \frac{A^{k}}{M} & (1+\delta t D) & 0 \\
0 & 0 & 0 & -\delta t & 1
\end{array}\right]
$$

$$
W^{j+1}=\left[\begin{array}{lllll}
\eta^{j+1, k^{\prime}} & \phi^{j+1} & p_{s}^{j+1, k} & h_{t}^{j+1} & h^{j+1}
\end{array}\right]^{\mathrm{T}}
$$

Superindex $k \longmapsto$ we kept the first $k$ columns Superindex $k^{\prime} \longrightarrow$ we kept all but the first $k$ columns

## Numerical Implementation

$$
\boldsymbol{Q}_{k} W_{k}^{W_{k}^{j+1}}=F_{k}^{j},
$$

## Numerical Implementation

$$
\begin{gathered}
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}\right)^{\mathrm{T}}-\delta t\left\{\frac{1}{F r} \boldsymbol{r}^{\boldsymbol{k}}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{I}^{k}\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\} \\
h_{t}^{j}-\delta t \frac{1}{F r}
\end{gathered}
$$

## Numerical Implementation

$$
\begin{gathered}
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
F_{k}^{j}=\left[\begin{array}{c}
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}-\delta t\left\{\frac{1}{F r} \boldsymbol{l}^{k}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{l}^{k}\left(\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\}\right.\right. \\
h_{t}^{j}-\delta t \frac{1}{F r} \\
h^{j}
\end{array}\right. \\
a_{k}=\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)[1,1, \ldots, 1]^{\mathrm{T}},
\end{gathered}
$$

## Numerical Implementation

$$
\begin{aligned}
& \boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
& \begin{array}{c}
F_{k}^{j}=\left[\begin{array}{c}
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}\right)^{\mathrm{T}}-\delta t\left\{\frac{1}{F r} \boldsymbol{l}^{k}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{l}^{k}\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\} \\
h_{t}^{j}-\delta t \frac{1}{F r} \\
h^{j}
\end{array}\right. \\
a_{k}=\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)[1,1, \ldots, 1]^{\mathrm{T}}, \quad b_{k}=\delta t\left(\frac{1}{F r} \boldsymbol{l}^{k}[1,1, \ldots, 1]^{\mathrm{T}}-\frac{1}{W e} c_{k}^{\mathrm{T}}\right),
\end{array}
\end{aligned}
$$

## Numerical Implementation

$$
\begin{gathered}
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
F_{k}^{j}=\left[\begin{array}{c}
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}\right)^{\mathrm{T}}-\delta t\left\{\frac{1}{F r} \boldsymbol{l}^{k}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{l}^{k}\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\} \\
h_{t}^{j}-\delta t \frac{1}{F r} \\
h^{j}
\end{array}\right. \\
a_{k}=\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)[1,1, \ldots, 1]^{\mathrm{T}}, \quad b_{k}=\delta t\left(\frac{1}{F r} \boldsymbol{l}^{k}[1,1, \ldots, 1]^{\mathrm{T}}-\frac{1}{W e} c_{k}^{\mathrm{T}}\right), \\
c_{k}=\frac{2 k-1}{2 k(\delta r)^{2}} e_{k+1}
\end{gathered}
$$

## Numerical Implementation

$$
\begin{gathered}
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
F_{k}^{j}=\left[\begin{array}{c}
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}\right)^{\mathrm{T}}-\delta t\left\{\frac{1}{F r} \boldsymbol{l}^{k}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{l}^{k}\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\} \\
h_{t}^{j}-\delta t \frac{1}{F r} \\
h^{j}
\end{array}\right. \\
a_{k}=\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)[1,1, \ldots, 1]^{\mathrm{T}}, \quad b_{k}=\delta t\left(\frac{1}{F r} \boldsymbol{r}^{k}[1,1, \ldots, 1]^{\mathrm{T}}-\frac{1}{W e} c_{k}^{\mathrm{T}}\right), \\
c_{k}=\frac{2 k-1}{2 k(\delta r)^{2}} e_{k+1}
\end{gathered}
$$

Superindex $k$

## Numerical Implementation

$$
\left.\begin{array}{c}
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
F_{k}^{j}=\left[\begin{array}{c}
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}\right)^{\mathrm{T}}-\delta t\left\{\frac{1}{F r} \boldsymbol{l}^{k}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{l}^{k}\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\} \\
h_{t}^{j}-\delta t \frac{1}{F r} \\
h^{j} \\
a_{k}=\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)
\end{array}\right] \\
{[1,1, \ldots, 1]^{\mathrm{T}}, \quad b_{k}=\delta t\left(\frac{1}{F r} \boldsymbol{r}^{k}[1,1, \ldots, 1]^{\mathrm{T}}-\frac{1}{W e} c_{k}^{\mathrm{T}}\right),}
\end{array}\right]
$$

Superindex $k \longmapsto$ we kept the first $k$ columns

## Numerical Implementation

$$
\begin{gathered}
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
F_{k}^{j}=\left[\begin{array}{c}
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}\right)^{\mathrm{T}}-\delta t\left\{\frac{1}{F r} \boldsymbol{l}^{k}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{l}^{k}\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\} \\
h_{t}^{j}-\delta t \frac{1}{F r} \\
h^{j}
\end{array}\right. \\
a_{k}=\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)[1,1, \ldots, 1]^{\mathrm{T}}, \quad b_{k}=\delta t\left(\frac{1}{F r} \boldsymbol{l}^{k}[1,1, \ldots, 1]^{\mathrm{T}}-\frac{1}{W e} c_{k}^{\mathrm{T}}\right), \\
c_{k}=\frac{2 k-1}{2 k(\delta r)^{2}} e_{k+1}
\end{gathered}
$$

Superindex $k \longmapsto$ we kept the first $k$ columns Superindex $k^{\prime}$

## Numerical Implementation

$$
\begin{gathered}
\boldsymbol{Q}_{k} W_{k}^{j+1}=F_{k}^{j}, \\
=\left[\begin{array}{c}
\left(\eta^{j}\right)^{\mathrm{T}}-\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)\left(z_{s}^{k}\right)^{\mathrm{T}} \\
\left(\phi^{j}\right)^{\mathrm{T}}-\delta t\left\{\frac{1}{F r} \boldsymbol{l}^{k}\left(z_{s}^{k}\right)^{\mathrm{T}}-\frac{1}{W e}\left(\boldsymbol{l}^{k}\left(\left(\kappa z_{s}\right)^{k}\right)^{\mathrm{T}}+z_{s}((k-1) \delta r) c_{k}^{\mathrm{T}}\right)\right\} \\
h_{t}^{j}-\delta t \frac{1}{F r} \\
h^{j}
\end{array}\right. \\
a_{k}=\left(\boldsymbol{l}^{k}-\frac{2 \delta t}{R e} \Delta_{H}^{k}\right)[1,1, \ldots, 1]^{\mathrm{T}}, \quad b_{k}=\delta t\left(\frac{1}{F r} r^{k}[1,1, \ldots, 1]^{\mathrm{T}}-\frac{1}{W e} c_{k}^{\mathrm{T}}\right), \\
c_{k}=\frac{2 k-1}{2 k(\delta r)^{2}} e_{k+1}
\end{gathered}
$$

Superindex $k \longmapsto$ we kept the first $k$ columns Superindex $k^{\prime} \longrightarrow$ we kept all but the first $k$ columns

We have a closed system of equations for each contact area that we test

We have a closed system of equations for each contact area that we test


We have a closed system of equations for each contact area that we test



We have a closed system of equations for each contact area that we test


## Experiment



## Experiment



Simulation


## Experiment



Experiment by Dan Harris (Brown)

## Droplets on a Shaking Free Surface

## Droplets on a Shaking Free Surface

## Droplets on a Shaking Free Surface

Each bounce triggers new waves

## Droplets on a Shaking Free Surface

Each bounce triggers new waves
Waves determine following bounces

## Droplets on a Shaking Free Surface

## Droplets on a Shaking Free Surface

This is a non-linear, non-smooth dynamical system

## Droplets on a Shaking Free Surface

$0.4 \mathrm{~mm}<D<1 \mathrm{~mm}$

This is a non-linear, non-smooth dynamical system

## Droplets on a Shaking Free Surface

$0.4 \mathrm{~mm}<D<1 \mathrm{~mm}$

$$
\lambda \approx 5 \mathrm{~mm}
$$

This is a non-linear, non-smooth dynamical system

## Droplets on a Shaking Free Surface

$0.4 \mathrm{~mm}<D<1 \mathrm{~mm}$
$\lambda \approx 5 \mathrm{~mm}$
$V_{z} \approx 10 \mathrm{~cm} / \mathrm{s}$

This is a non-linear, non-smooth dynamical system

## Droplets on a Shaking Free Surface

## $0.4 \mathrm{~mm}<D<1 \mathrm{~mm}$

$\lambda \approx 5 \mathrm{~mm}$
$V_{z} \approx 10 \mathrm{~cm} / \mathrm{s}$
$e \approx 2 \mu \mathrm{~m}$

This is a non-linear, non-smooth dynamical system

## Droplets on a Shaking Free Surface

$0.4 \mathrm{~mm}<D<1 \mathrm{~mm}$
$\lambda \approx 5 \mathrm{~mm}$
$V_{z} \approx 10 \mathrm{~cm} / \mathrm{s}$
$e \approx 2 \mu \mathrm{~m}$
$f=40 \mathrm{~Hz}$

This is a non-linear, non-smooth dynamical system

## Droplets on a Shaking Free Surface

$0.4 \mathrm{~mm}<D<1 \mathrm{~mm}$
$\lambda \approx 5 \mathrm{~mm}$
$V_{z} \approx 10 \mathrm{~cm} / \mathrm{s}$
$e \approx 2 \mu \mathrm{~m}$
$f=40 \mathrm{~Hz}$
$A \approx 10 \mu \mathrm{~m}$

This is a non-linear, non-smooth dynamical system

## $t / T_{f}=0.00$



## $t / T_{f}=0.00$



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## Non-wetting impact of a sphere onto a bath and its application to bouncing droplets

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(Received 9 December 2016; revised 26 April 2017; accepted 6 June 2017)

## Droplets Walking on a Free Surface

Video: Dan Harris \& John Bush

## Droplets Walking on a Free Surface

Video: Dan Harris \& John Bush

## Droplets Walking on a Free Surface

## Walker

Video: Dan Harris \& John Bush

## Droplets Walking on a Free Surface

## Walker

$$
V_{x} \lesssim 1.5 \mathrm{~cm} / \mathrm{s}
$$

Video: Dan Harris \& John Bush

## Droplets Walking on a Free Surface

$$
V_{z} \approx 10 \mathrm{~cm} / \mathrm{s}
$$

## Walker

$$
V_{x} \lesssim 1.5 \mathrm{~cm} / \mathrm{s}
$$

Video: Dan Harris \& John Bush

## Droplets Walking on a Free Surface

$$
V_{z} \approx 10 \mathrm{~cm} / \mathrm{s} \quad 0.4 \mathrm{~mm}<D<1 \mathrm{~mm}
$$

$$
V_{x} \lesssim 1.5 \mathrm{~cm} / \mathrm{s}
$$

Video: Dan Harris \& John Bush

## Droplets Walking on a Free Surface

$$
V_{z} \approx 10 \mathrm{~cm} / \mathrm{s}
$$

# $0.4 \mathrm{~mm}<D<1 \mathrm{~mm}$ 

$\lambda \approx 5 \mathrm{~mm}$

$$
V_{x} \lesssim 1.5 \mathrm{~cm} / \mathrm{s}
$$

Video: Dan Harris \& John Bush


Modelling a Walker

Modelling a Walker


Modelling a Walker

Modelling a Walker


Modelling a Walker


Modelling a Walker


Modelling a Walker


## Modelling a Walker



## Modelling a Walker



## Modelling a Walker



## Modelling a Walker



# Quasi-normal free-surface impacts, capillary rebounds and application to Faraday walkers 

C. A. Galeano-Rios ${ }^{1, \dagger}$, P. A. Milewski ${ }^{1}$ and J.-M. Vanden-Broeck ${ }^{2}$
${ }^{1}$ Department of Mathematical Sciences, University of Bath, Bath BA2 7AY, UK
${ }^{2}$ Department of Mathematics, University College London, London WC1E 6BT, UK
(Received 4 December 2018; revised 9 May 2019; accepted 10 May 2019)

$$
\begin{aligned}
& \gamma / \gamma_{F}=0.22 \\
& \text { Large: }(1,1), \text { Small: }(1,1)
\end{aligned}
$$ Strobed at 80 Hz



Video from Galeano Rios et al 2018 Ratcheting droplet pairs (Chaos 2018)

$$
\begin{aligned}
& \gamma / \gamma_{F}=0.22 \\
& \text { Large: }(1,1), \text { Small: }(1,1)
\end{aligned}
$$ Strobed at 80 Hz



Video from Galeano Rios et al 2018 Ratcheting droplet pairs (Chaos 2018)


# Video: Miles Couchman 

Video from Galeano Rios et al 2018 Ratcheting droplet pairs (Chaos 2018)


# Video: Miles Couchman 

Video from Galeano Rios et al 2018 Ratcheting droplet pairs (Chaos 2018)


# Video: Miles Couchman 

Video from Galeano Rios et al 2018 Ratcheting droplet pairs (Chaos 2018)


# Video: Miles Couchman 

Video from Galeano Rios et al 2018 Ratcheting droplet pairs (Chaos 2018)

Ratcheting droplet pairs
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## Thanks for your attention

## Thanks for your attention

