Linear diffraction modelling for multi-float platforms for wave energy: from operational to extreme waves

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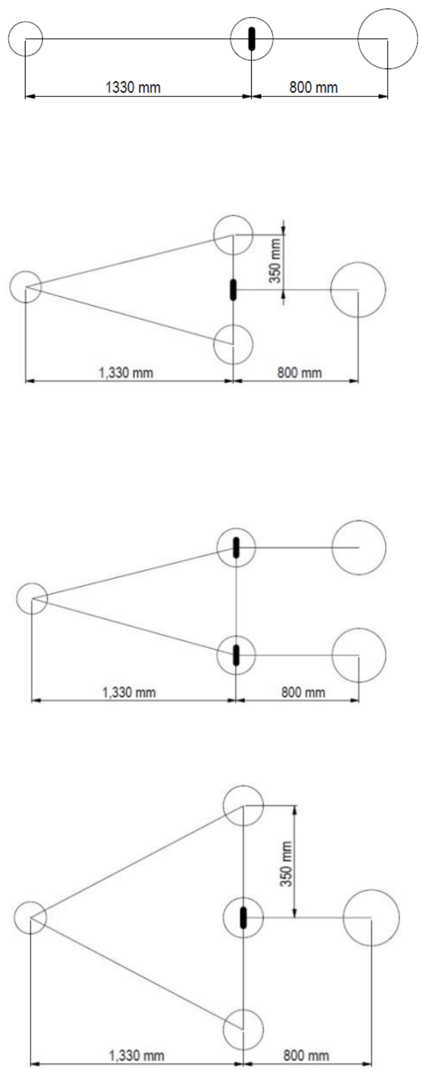
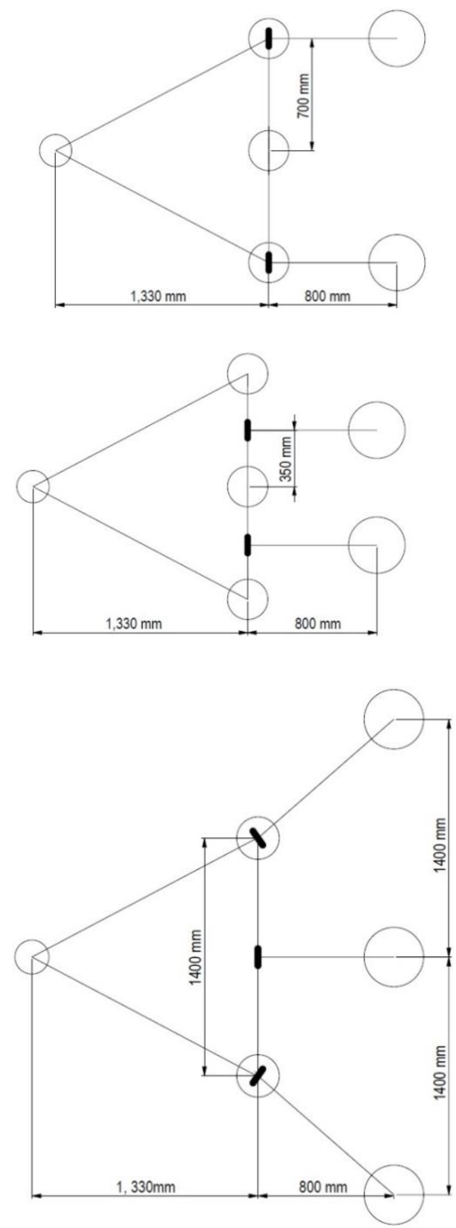
Outline of talk

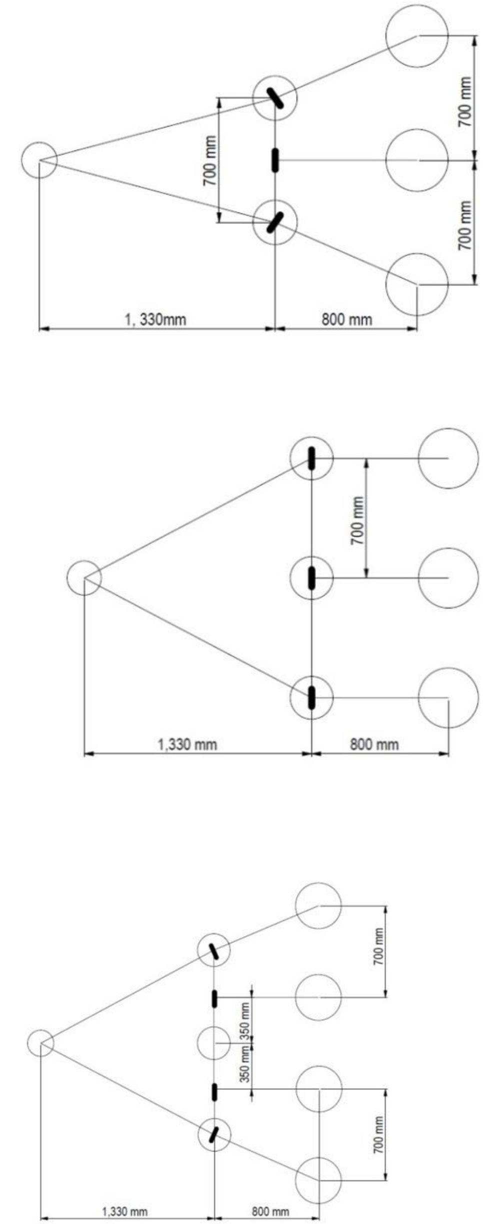
- WEC background
- Latest M4 configurations – LCOE
- Principles of wave loading
- Multi-float model
- Results – operational and extreme waves - response, accns, mooring forces
- Future developments – control, hybrids, big capacity

M4 wave energy 2016

Configurations with linear modelling







Stansby,P., et al 2017 Applied Ocean Research , 68 53–64.

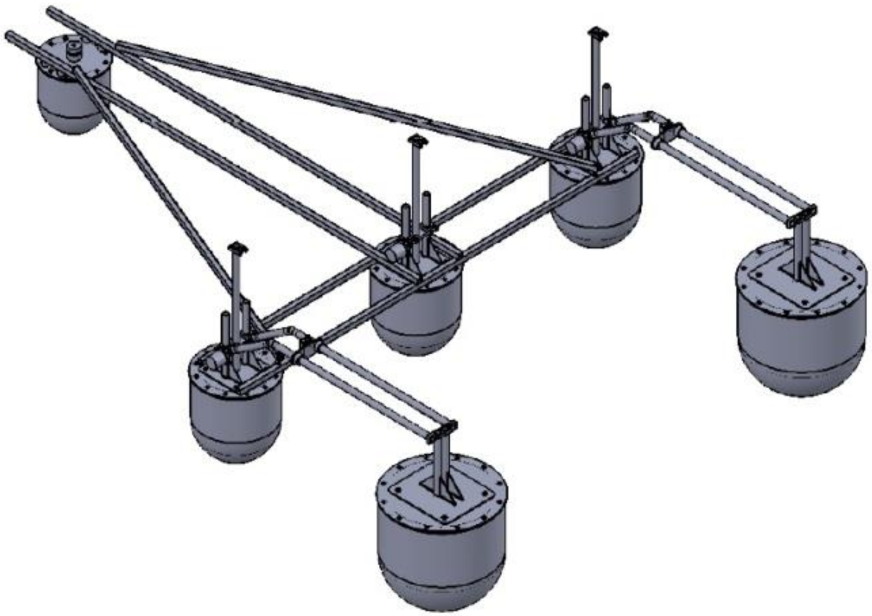
Wavehub, Cornwall

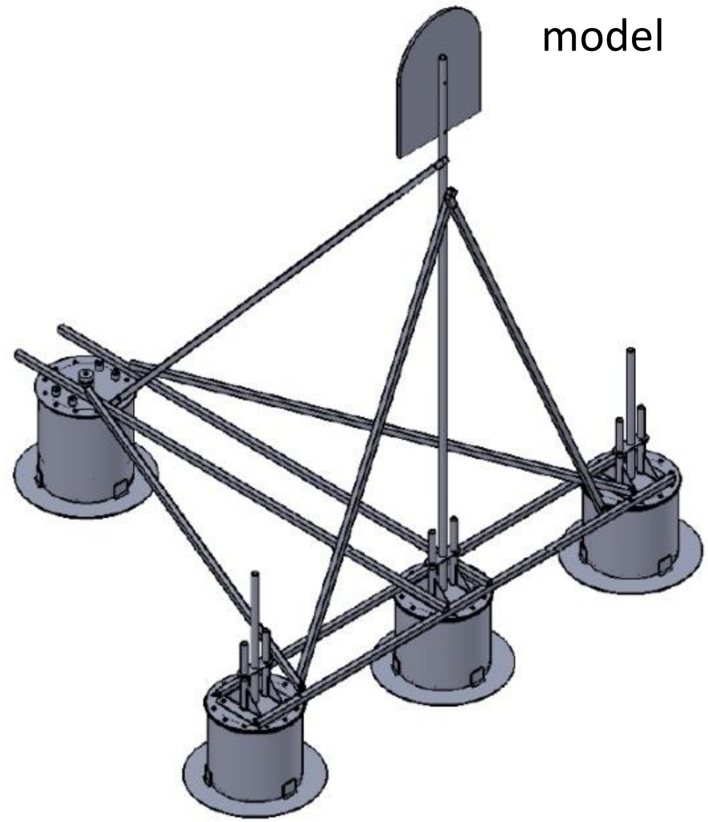
WaveHub, Cornwall, UK with scatter diagram from Van Nieuwkoop et al (2010)							
config	Optimum LSF	Average power [kW]	Annual energy yield [MWh]	Rated Power [kW]	Cost[M£]/MW	Total cost [M£]	LCoE [£/kWh]
3fl_111	50	281	2466	844	2.547	1.737	0.183
4fl_121	50	435	3815	1306	2.138	2.256	0.154
5fl_131	50	482	4226	1447	2.374	2.775	0.171
5fl_122a	50	592	5186	1776	2.218	3.181	0.159
5fl_122b	50	564	4948	1694	2.325	3.181	0.167
6fl_132a	**50**	**713**	**6249**	**2140**	**2.141**	**3.700**	**0.154**
6fl_132b	50	660	5786	1981	2.312	3.700	0.166
6fl_123a	60	900	7887	2701	2.711	5.913	0.195
6fl_123b	50	619	5428	1859	2.735	4.106	0.196
7fl_133	50	758	6642	2274	2.518	4.625	0.181
8fl_134	60	1246	10915	3738	2.648	7.992	0.190

Death Coast, N Spain

Death Cost, Spain with scatter diagram from Iglesias and Caraballo (2009)							
config	Optimum LSF	Average power [kW]	Annual energy yield [MWh]	Rated Power [kW]	Cost[M£]/MW	Total cost [M£]	LCoE [£/kWh]
3fl_111	120	1915	16778	5745	2.156	10.006	0.155
4fl_121	120	2948	25832	8846	1.819	12.995	0.131
5fl_131	120	3264	28596	9793	2.021	15.985	0.145
5fl_122a	120	4027	35282	12083	1.878	18.323	0.135
5fl_122b	120	3871	33914	11614	1.954	18.323	0.140
6fl_132a	**120**	**4813**	**42168**	**14441**	**1.828**	**21.313**	**0.131**
6fl_132b	120	4476	39210	13428	1.966	21.313	0.141
6fl_123a	120	4315	37803	12946	2.262	23.652	0.163
6fl_123b	120	4225	37017	12677	2.310	23.652	0.166
7fl_133	120	5212	45663	15638	2.110	26.641	0.152
8fl_134	120	5944	52075	17833	2.220	31.969	0.159

Marinet2 test geometries



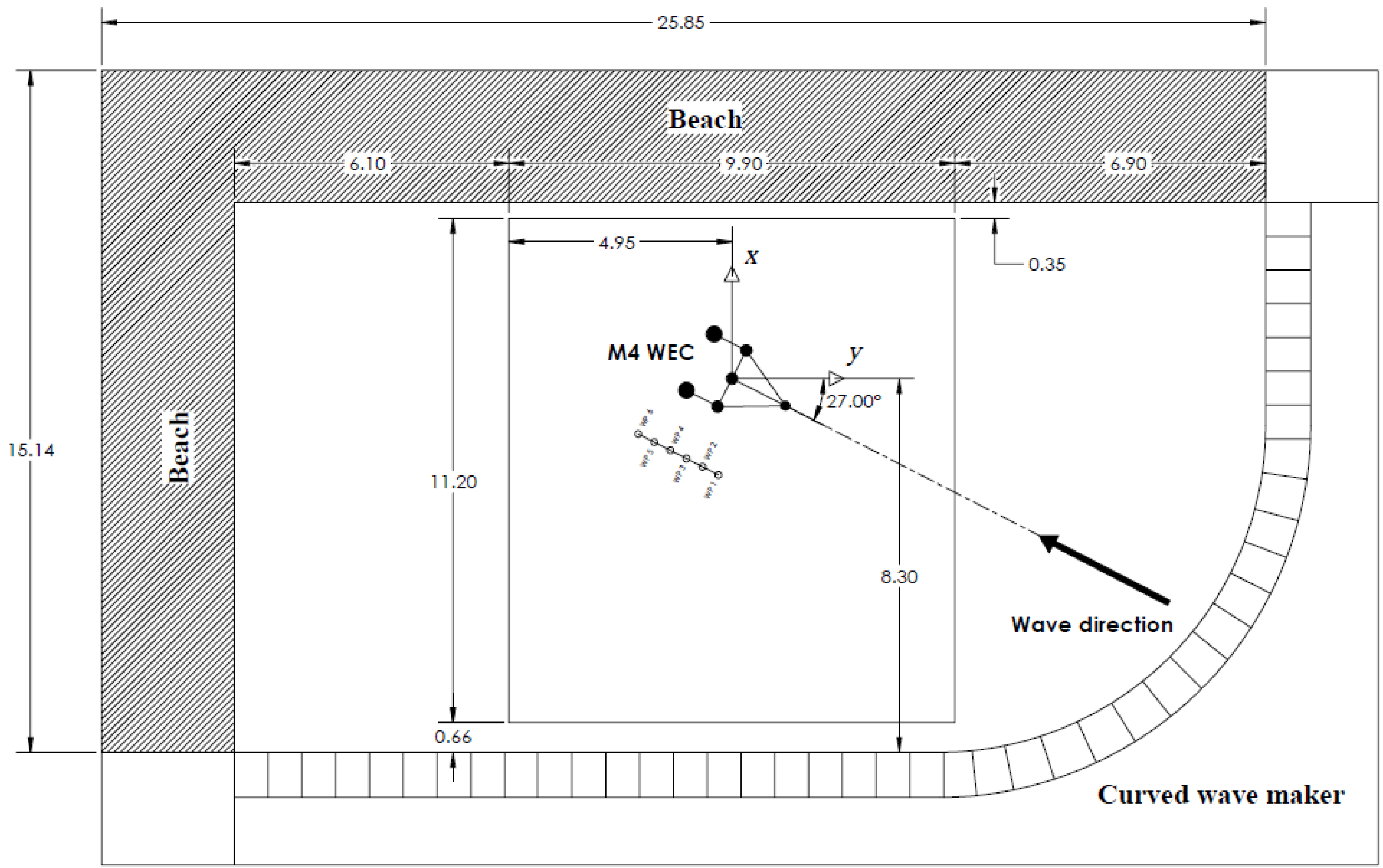


Power as predicted?
Mooring forces ?

Hub accns < 4 m/s^2
Draft < 10 m

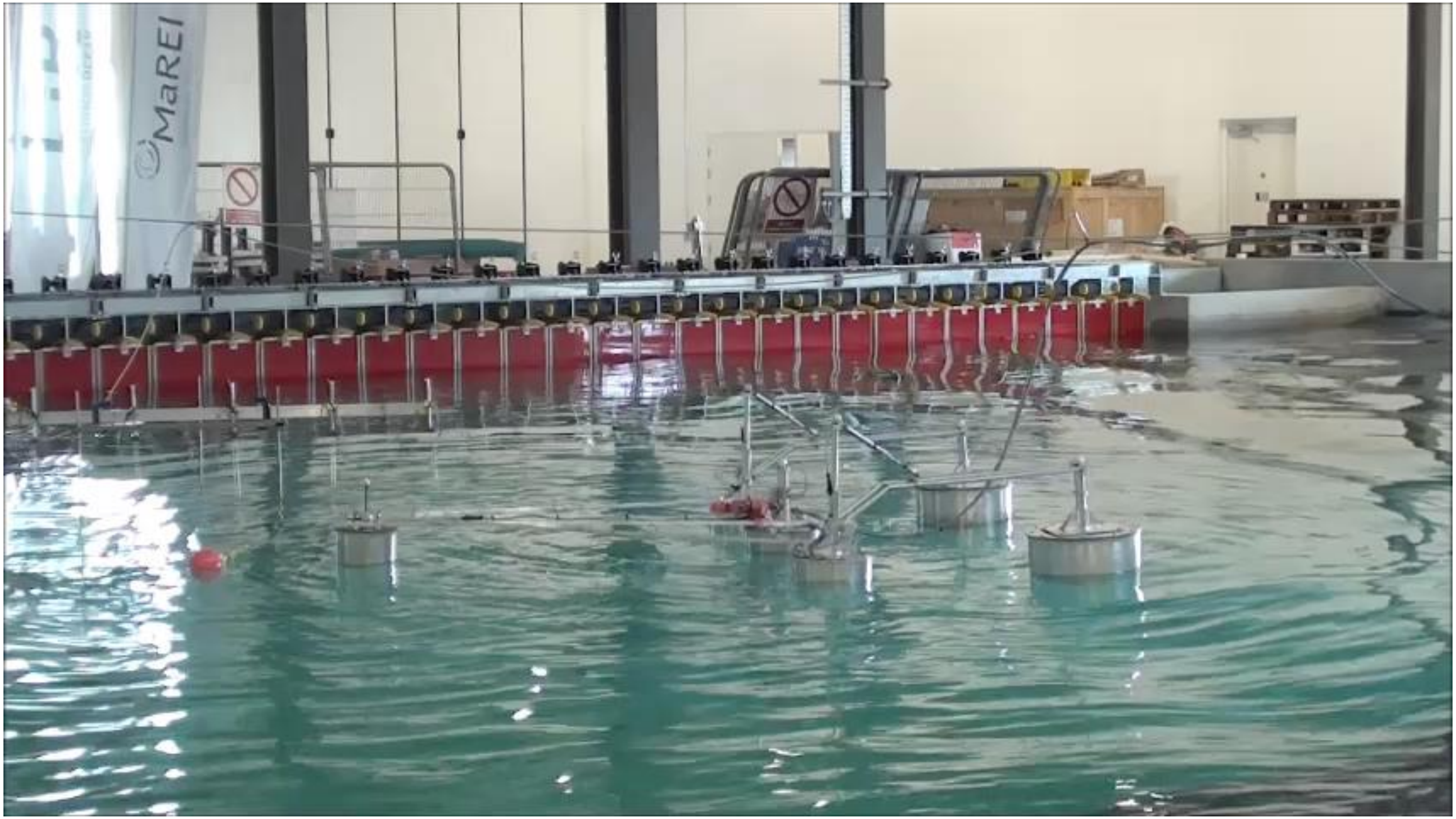
Carpintero Moreno, E., Stansby, P.K. 2019 Renewable and Sustainable Energy Reviews,104, 307–318

Lir ocean wave basin UCC



The team

6 float M4



Linear diffraction principle

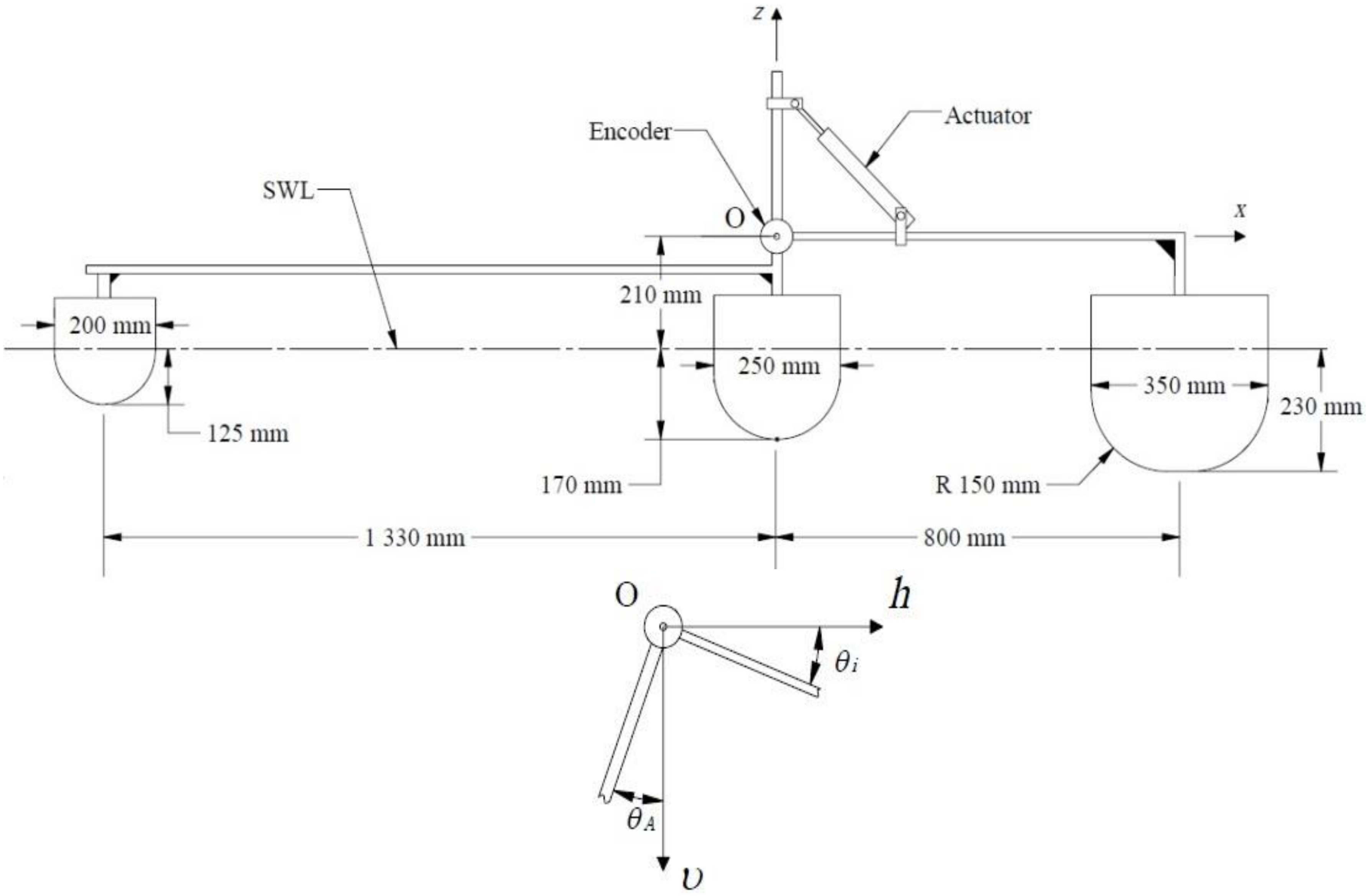
Force =
Diffraction/excitation force (fixed body) +
added mass force (body acceleration) +
radiation damping (body velocity) +
restoring (hydrostatic) force +
drag (body velocity, nonlinear) +
mechanical damping (linear or nonlinear) +
wind turbine forces when applied (nonlinear)

With coefficients from WAMIT (potential flow panel code)

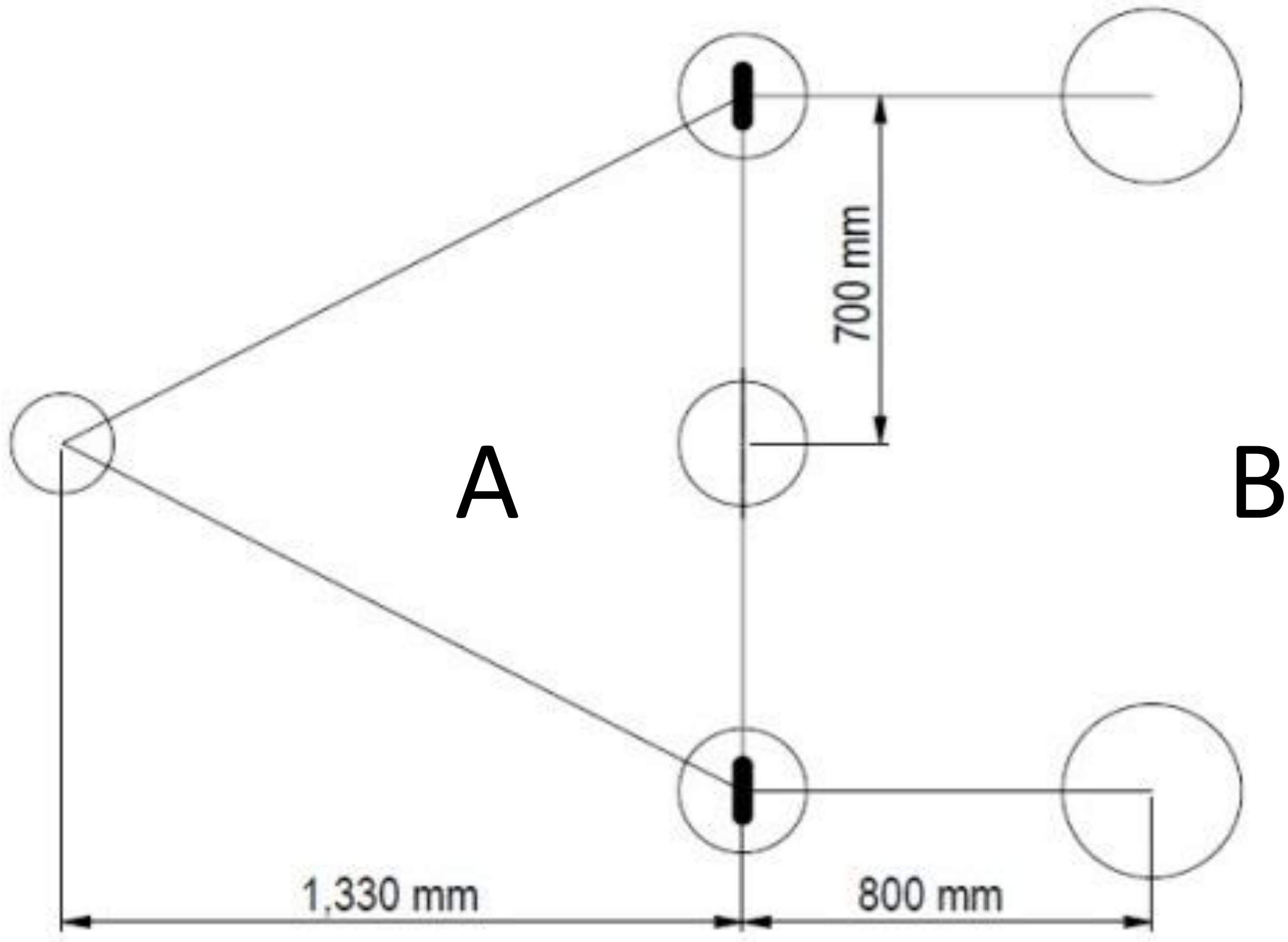
Input for multi-body model

In addition 2^{nd} order mean forces

lab model elevation



Multi-floats analysed - example



For body A taking moments about O

$$-\sum_{i=1}^{n_A} m_i\, v_i\, \ddot{x}_i \;-\; \sum_{i=1}^{n_A} m_i\, h_i\, \ddot{z}_i \;+\; I_A \ddot{\theta}_A \;=\; M_{mech} + \sum_{i=1}^{n_A} M_i - \sum_{i=1}^{n_A} h_i\, V_i - \sum_{i=1}^{n_A} v_i\, H_i \;+ v_M H_M - h_M V_M$$

For body B for each float taking moments about O

$$- m_i\, v_i\, \ddot{x}_i - \; m_i\, h_i\, \ddot{z}_i \;+\; I_i \ddot{\theta}_i \;=\; -M_{mech\, i} + M_i \;-\; h_i\, V_i \;-\; v_i\, H_i$$

where for linear PTO

$M_{mec\,h\;i} = -B_{mec\,h}\,\dot{\theta}_{r\;i}$, $M_{mec\,h} = \sum_{1+n_A}^{n} M_{mec\,h\;i}$ and $\theta_{r\;i} = \theta_A - \theta_i$

For the whole system there is no net force or moment on the hinge

$$\sum_{i=1}^{N} m_i \, \ddot{x}_i = \sum_{i=1}^{N} H_i - H_M$$

$$\sum_{i=1}^{N} m_i \, \ddot{z}_i = \sum_{i=1}^{N} V_i - V_M$$

$$x_i = x_O - v_i \, \theta_i$$
$$z_i = z_O - h_i \, \theta_i - t_i \, \theta_R$$
$$y_i = y_O - v_i \, \theta_R$$

Moments about O give

A:

$$\ddot{\theta}_A\left(\sum_{i=1}^{n_A} m_i\, {v_i}^2 \; + \sum_{i=1}^{n_A} m_i\, {h_i}^2 \; + \; I_A \;\right) =$$

$$\sum_{i=1}^{n_A} m_i\, v_i\, \ddot{x}_O + \sum_{i=1}^{n_A} m_i\, h_i\, \ddot{z}_O + \; M_{mech} + \sum_{i=1}^{n_A} M_i - \sum_{i=1}^{n_A} h_i\; V_i$$

$$- \sum_{i=1}^{n_A} v_i\, H_i \quad + \; v_M H_M \;\; - h_M V_M$$

B:

$$\ddot{\theta}_i\left(m_i\, {v_i}^2 + \; m_i\, {h_i}^2 + \; I_i\right)$$

$$= \; m_i\, v_i\, \ddot{x}_O + m_i\, h_i\, \ddot{z}_O \; - \; M_{mech\; i} + \; M_i - \; h_i\, V_i$$

$$- \; v_i\, H_i$$

Whole system

$$\ddot{x}_O \sum_{i=1}^{N} m_i = \sum_{i=1}^{N} H_i - H_M + \sum_{i=1}^{N} m_i\, v_i \ddot{\theta}_i$$

$$\ddot{z}_O \sum_{i=1}^{N} m_i = \sum_{i=1}^{N} V_i - V_M + \sum_{i=1}^{N} m_i\, h_i \ddot{\theta}_i + m_i t_i\, \ddot{\theta}_R$$

4 equations and 4 unknowns: $\ddot{\theta}_A, \ddot{\theta}_B, \ddot{x}_O, \ddot{z}_O$

Cummins convolution method for irregular waves

$$(m + A_{ij}^{\infty})\,\ddot{x}(t) + \int_{-\infty}^{t} L_{ij}(t-\tau)\,\dot{x}(\tau)\,\mathrm{d}\tau = f(t)$$

Impulse response functions pre-computed

$$L_{ij}(t) = \frac{2}{\pi}\int_{0}^{\infty} B_{ij}(\omega)\cos(\omega t)\,d\omega$$

Forces from WAMIT

WAMIT notation	
Body i	Mode number
Surge	1+6(i-1)
Heave	3+6(i-1)
Pitch	5+6(i-1)

$$M_i = M_{D\ 5+6(i-1)} - \sum_{j=1}^{n} A^{\infty}{}_{5+6(i-1),\,5+6(j-1)} \cdot \ddot{\theta}_j$$

$$- \sum_{j=1}^{n} \int_{-\infty}^{t} L_{5+6(i-1),5+6(j-1)}(t-\tau)\ \dot{\theta}_j(\tau)\ d\tau$$

$$- \sum_{j=1}^{n} A^{\infty}{}_{5+6(i-1),\,1+6(j-1)} \cdot \ddot{x}_j - \sum_{j=1}^{n} \int_{-\infty}^{t} L_{5+6(i-1),1+6(j-1)}(t-\tau)\ \dot{x}_j(\tau)\ d\tau$$

$$- \sum_{j=1}^{n} A^{\infty}{}_{5+6(i-1),\,3+6(j-1)} \cdot \ddot{z}_j - \sum_{j=1}^{n} \int_{-\infty}^{t} L_{5+6(i-1),3+6(j-1)}(t-\tau)\ \dot{z}_j\ d\tau$$

$$+ M_{rest\ i} + M_{drag\ i} + M_{mean\ i}$$

$$V_i = V_{D\ 3+6(i-1)} - \sum_{j=1}^{n} A^{\infty}{}_{3+6(i-1),\,5+6(j-1)} \cdot \ddot{\theta}_j$$
$$- \sum_{j=1}^{n} \int_{-\infty}^{t} L_{3+6(i-1),\,5+6(j-1)}(t-\tau)\ \dot{\theta}_j(\tau)\, d\tau$$
$$- \sum_{j=1}^{n} A^{\infty}{}_{3+6(i-1),\,1+6(j-1)} \cdot \ddot{x}_j - \sum_{j=1}^{n} \int_{-\infty}^{t} L_{3+6(i-1),1+6(j-1)}(t-\tau)\ \dot{x}_j(\tau)\, d\tau$$
$$- \sum_{j=1}^{n} A^{\infty}{}_{3+6(i-1),\,3+6(j-1)} \cdot \ddot{z}_j - \sum_{j=1}^{n} \int_{-\infty}^{t} L_{3+6(i-1),\,3+6(j-1)}(t-\tau)\ \dot{z}_j(\tau)\, d\tau$$
$$+ V_{rest\ i} + V_{drag\ i} + V_{mean\ i}$$

$$H_i = H_{D\ 1+6(i-1)} - \sum_{j=1}^{n} A^{\infty}{}_{1+6(i-1),\,5+6(j-1)} \cdot \ddot{\theta}_j$$
$$- \sum_{j=1}^{n} \int_{-\infty}^{t} L_{1+6(i-1),\,5+6(j-1)}(t-\tau)\ \dot{\theta}_j(\tau)\, d\tau - \sum_{j=1}^{n} A^{\infty}{}_{1+6(i-1),\,1+6(j-1)} \cdot \ddot{x}_j$$
$$- \sum_{j=1}^{n} \int_{-\infty}^{t} L_{1+6(i-1),\,1+6(j-1)}(t-\tau)\ \dot{x}_j(\tau)\, d\tau - \sum_{j=1}^{n} A^{\infty}{}_{1+6(i-1),\,3+6(j-1)} \cdot \ddot{z}_j$$
$$- \sum_{j=1}^{n} \int_{-\infty}^{t} L_{1+6(i-1),\,3+6(j-1)}(t-\tau)\ \dot{z}_j(\tau)\, d\tau$$
$$+ H_{drag\ i} + H_{mean\ i}$$

Restoring forces

$$V_{rest} = -\rho g \pi r^2 z$$

$$M_{rest} = -\rho g \pi \frac{r^4}{4} \theta$$

power = $\sum_{i=1+n_A}^{n} - M_{mech\,i}\ \theta_{r\,i}$

Mean horizontal forces due to :

2nd order difference freqs when equal (from WAMIT)

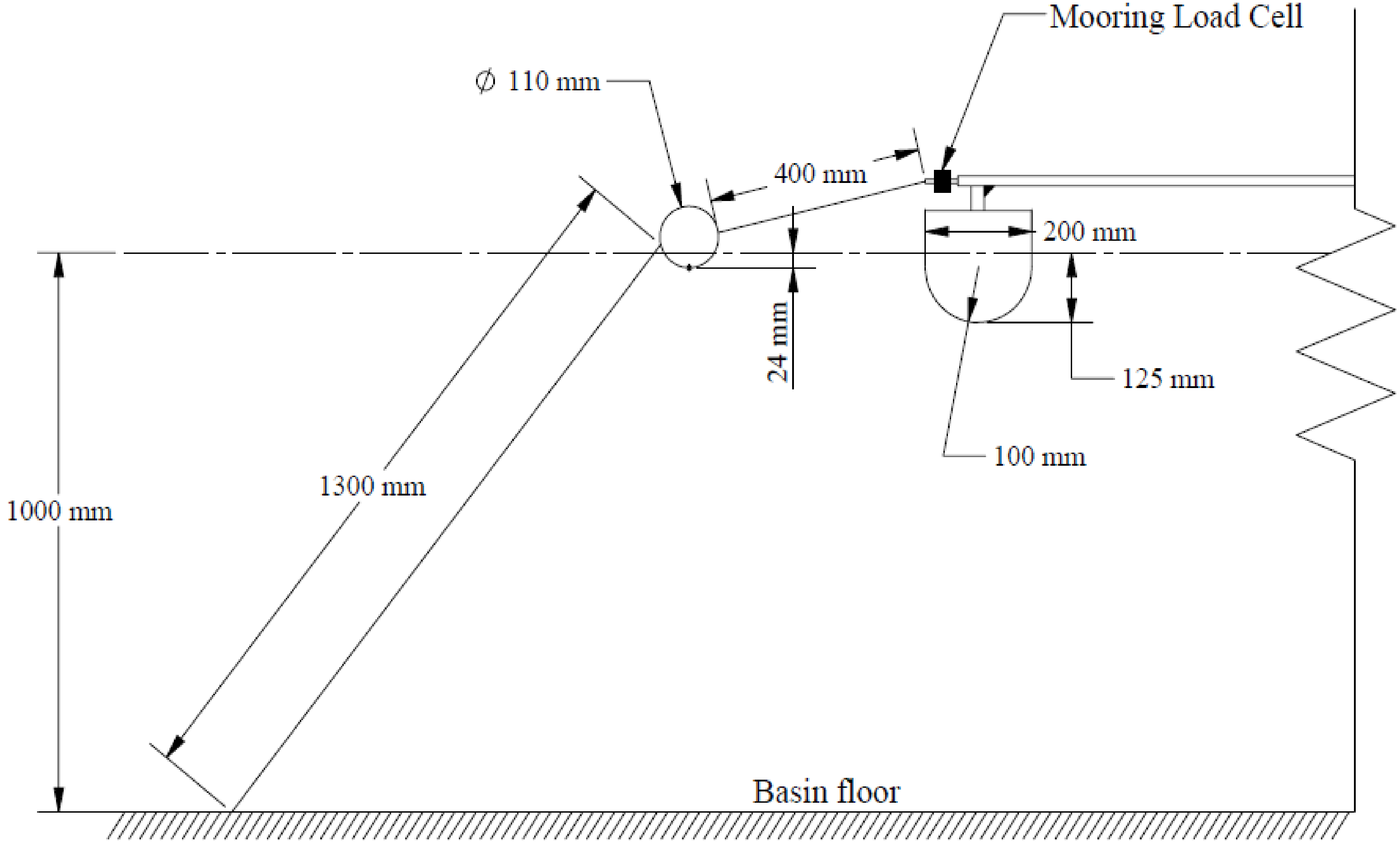
+ energy flux to radiate waves

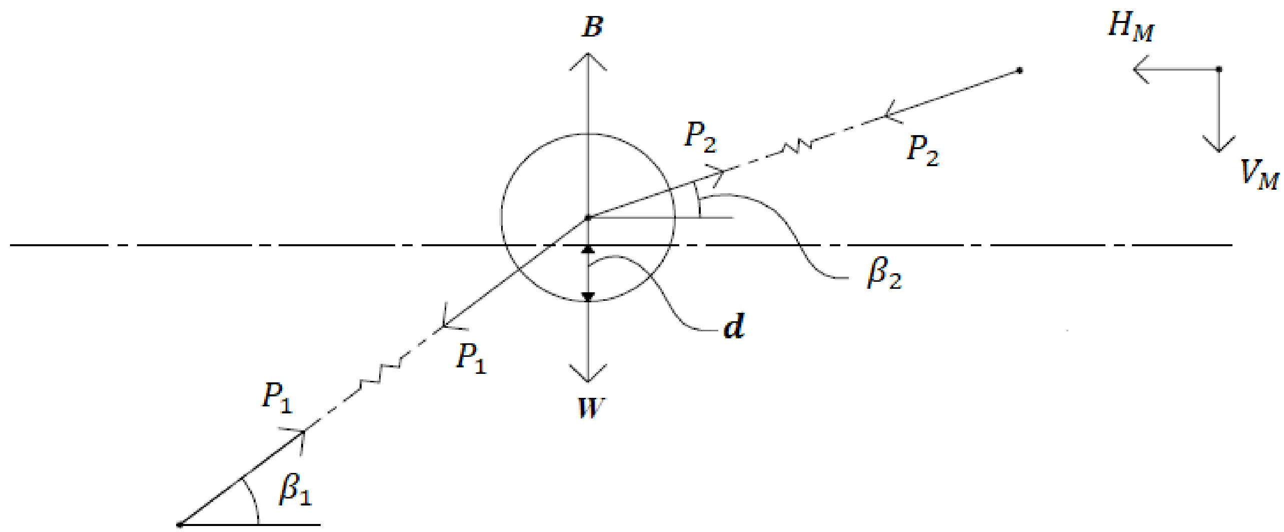
+ energy flux in mech damping

+ energy flux in drag damping

Average horizontal force x average wave speed = average energy flux (P)

Mooring fully coupled





$$H_M = P_2 \cos(\beta_2) \qquad V_M = P_2 \sin(\beta_2) \qquad P_2 \cos(\beta_2) = P_1 \cos(\beta_1) = H_M$$

$$B - W = P_1 \sin(\beta_1) - P_2 \sin(\beta_2) = H_M \left(\tan(\beta_1) - \tan(\beta_2)\right)$$

$$H_M = {(B - W)}\Big/{(\tan(\beta_1) - \tan(\beta_2))}$$

$$l_1 \cos(\beta_1) + l_2 \cos(\beta_2) = x_{b1} \qquad l_1 \sin(\beta_1) + l_2 \sin(\beta_2) = z_{b1}$$

$$d < 2r \;\; B = \rho\, g\, \pi \frac{d^2(3r - d)}{3} \qquad\qquad d > 2r \;\; B = 4\, \rho\, g\, \pi \frac{r^3}{3}$$

Beeman's time stepping

$$\theta_A^{n+1} = \theta_A^n + \dot{\theta}_A^n \Delta t + \ddot{\theta}_A^n \Delta t^2/2$$

$$\theta_B^{n+1} = \theta_B^n + \dot{\theta}_B^n \Delta t + \ddot{\theta}_B^n \Delta t^2/2$$

$$\dot{\theta}_A^{n+1} = \dot{\theta}_A^n + \ddot{\theta}_A^n \Delta t$$

$$\dot{\theta}_B^{n+1} = \dot{\theta}_B^n + \ddot{\theta}_B^n \Delta t$$

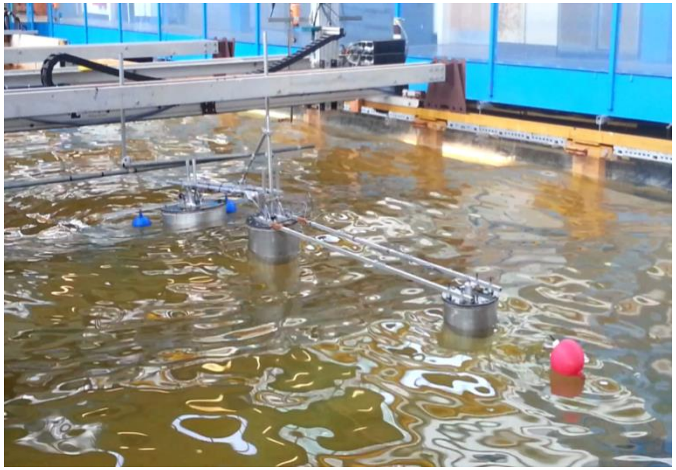
$$x_O^{n+1} = x_O^n + \dot{x}_O^n \Delta t + \ddot{x}_O^n \Delta t^2/2$$

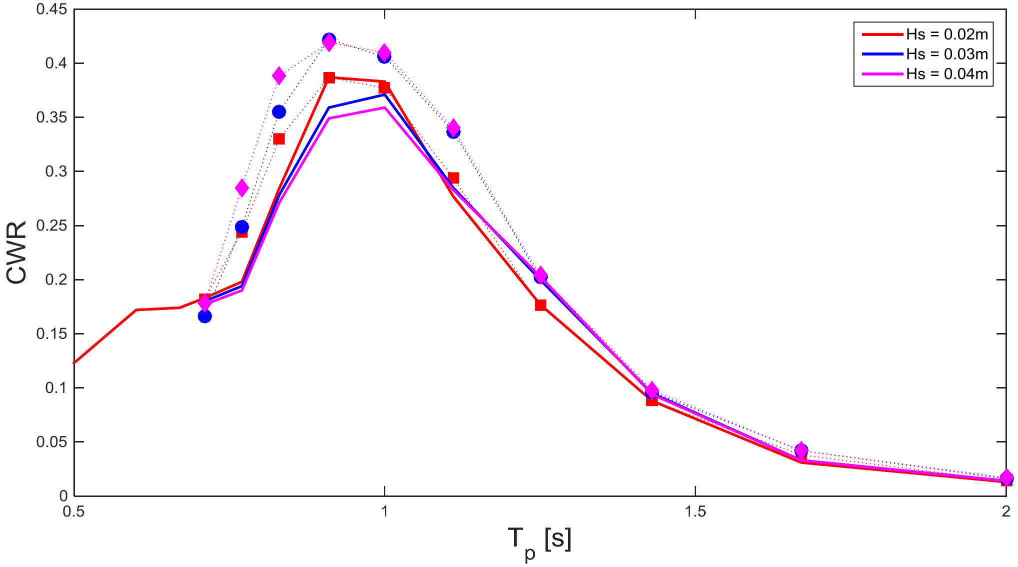
$$z_O^{n+1} = z_O^n + \dot{z}_O^n \Delta t + \ddot{z}_O^n \Delta t^2/2$$

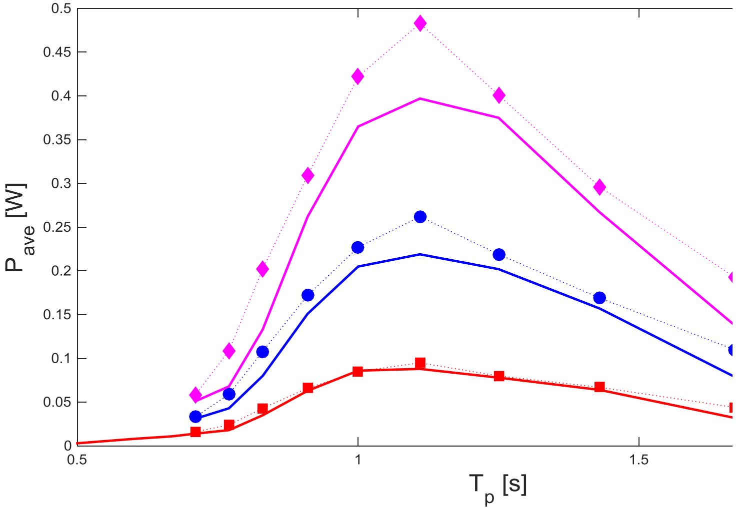
$$\dot{x}_O^{n+1} = \dot{x}_O^n + \ddot{x}_O^n \Delta t$$

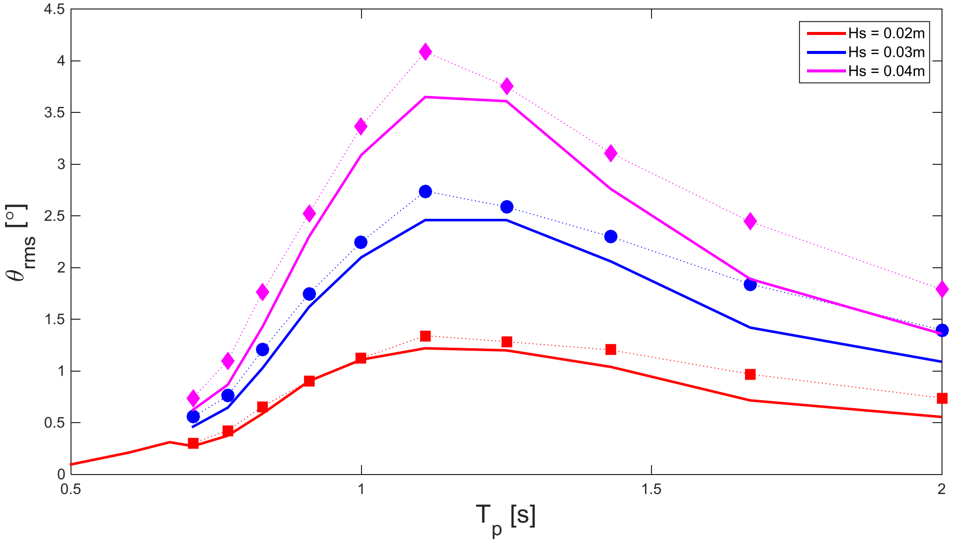
$$\dot{z}_O^{n+1} = \dot{z}_O^n + \ddot{z}_O^n \Delta t$$

Validation γ=3.3

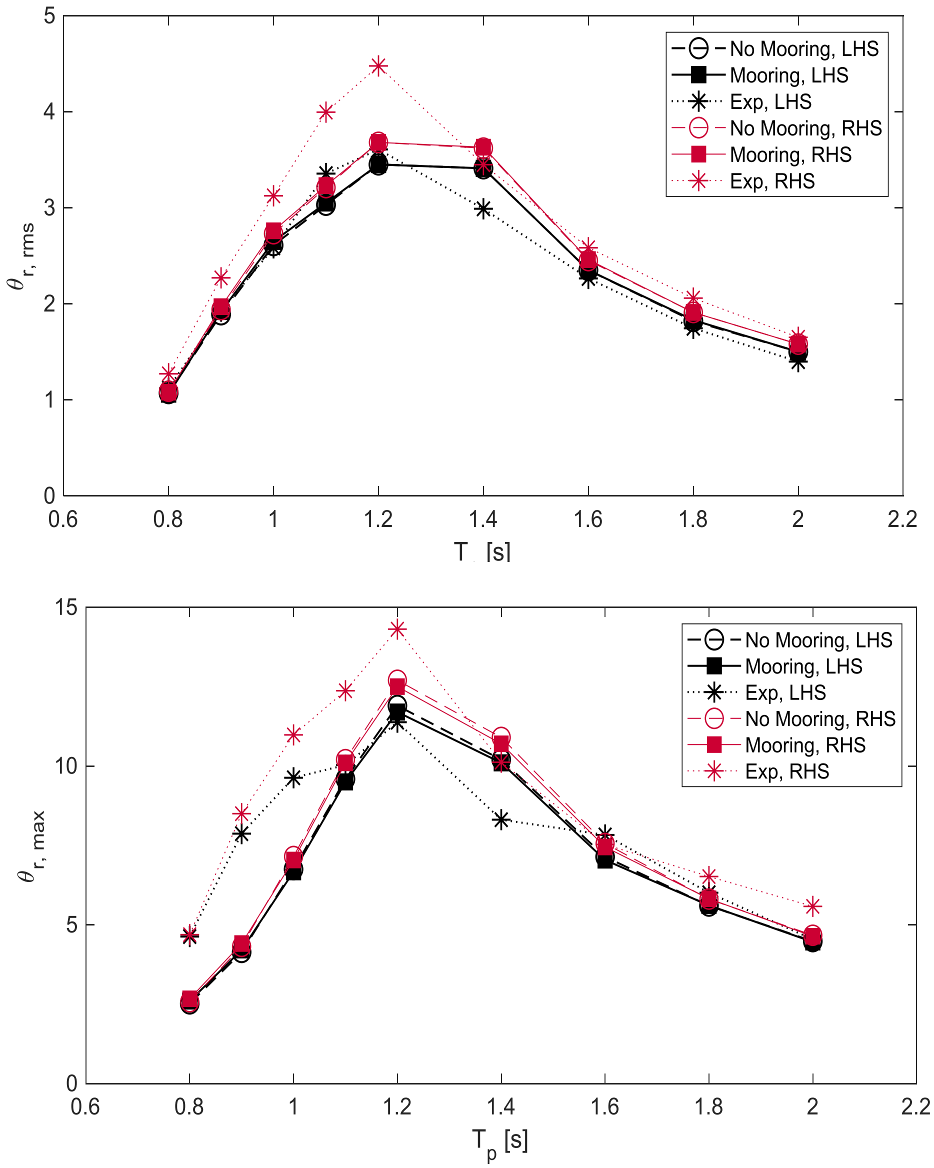




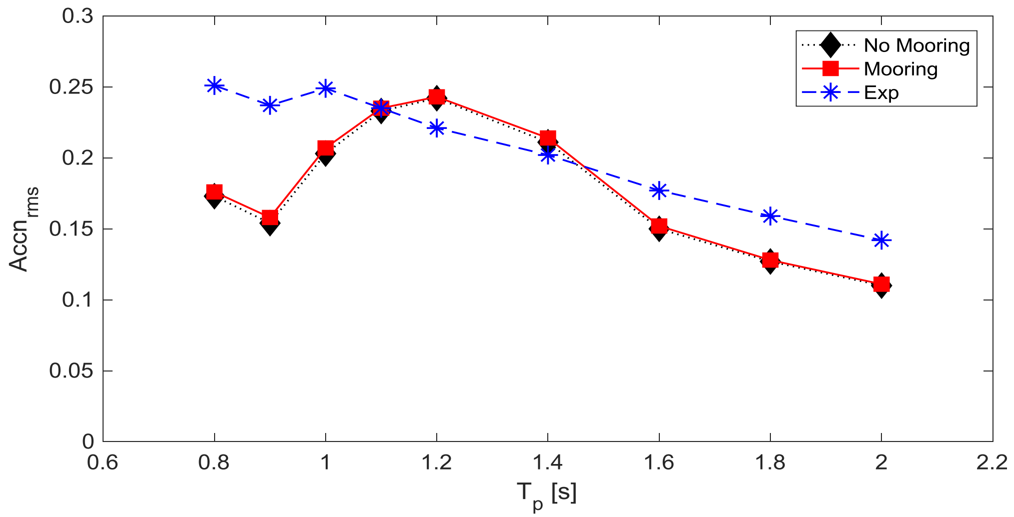


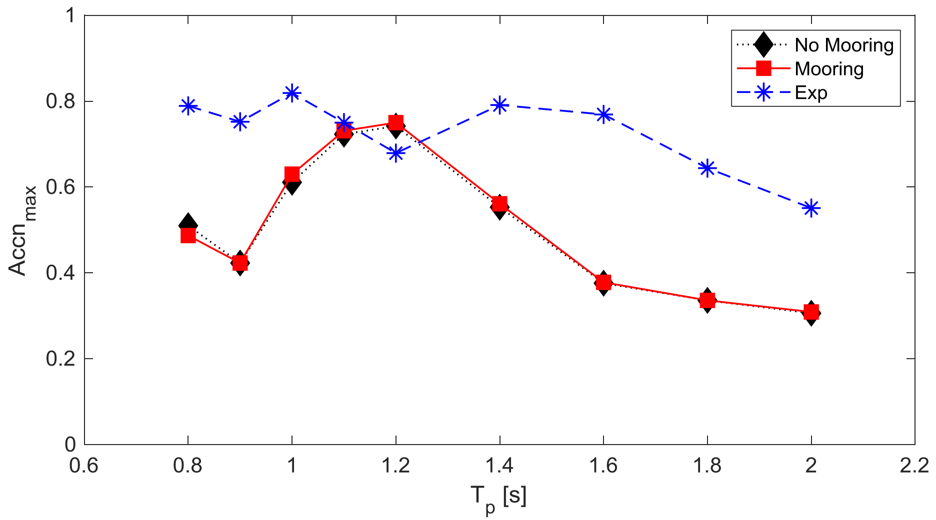


6 float WEC , angles (operational Hs)

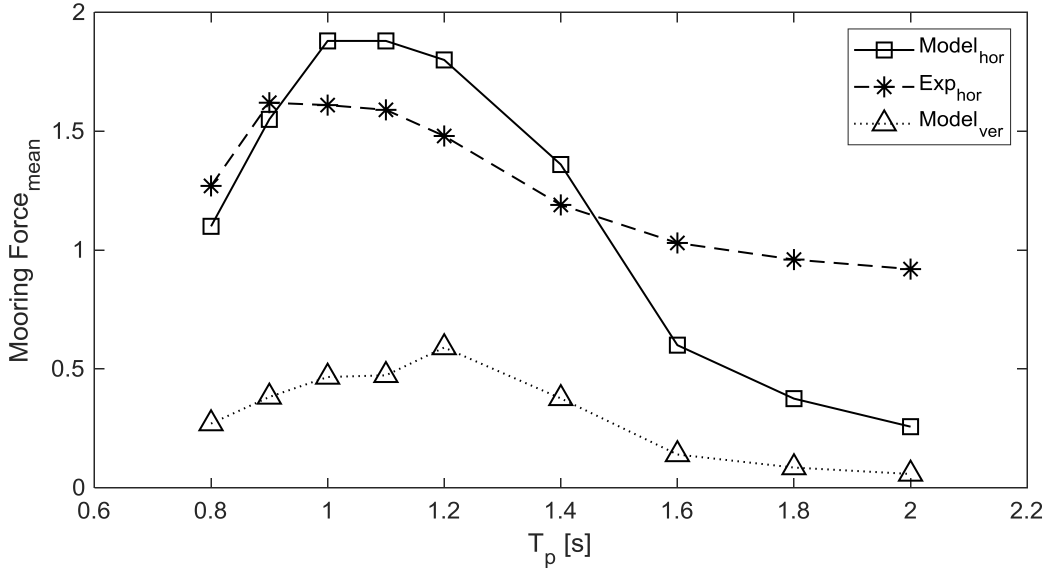


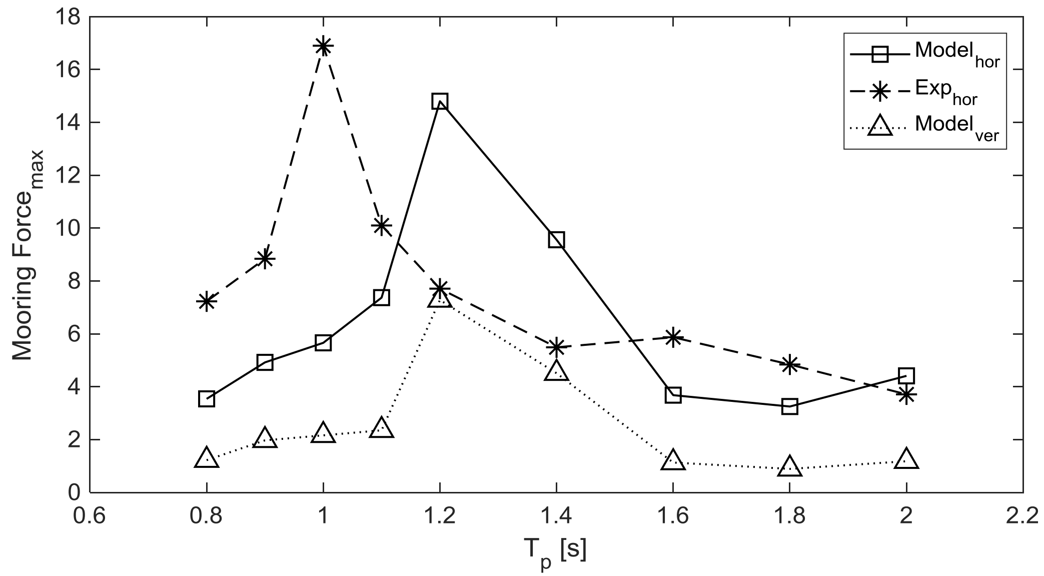
WEC accelerations at O, rms and max



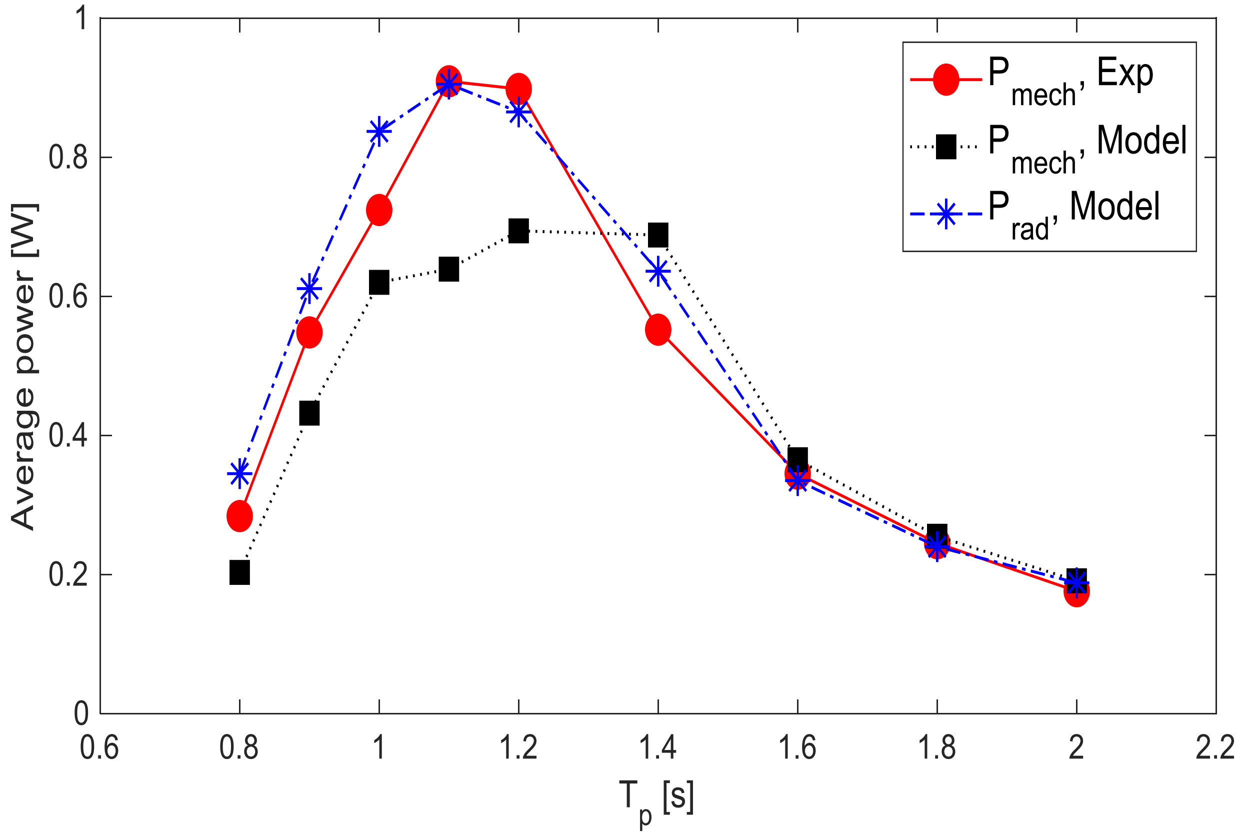


WEC mooring force, mean and max

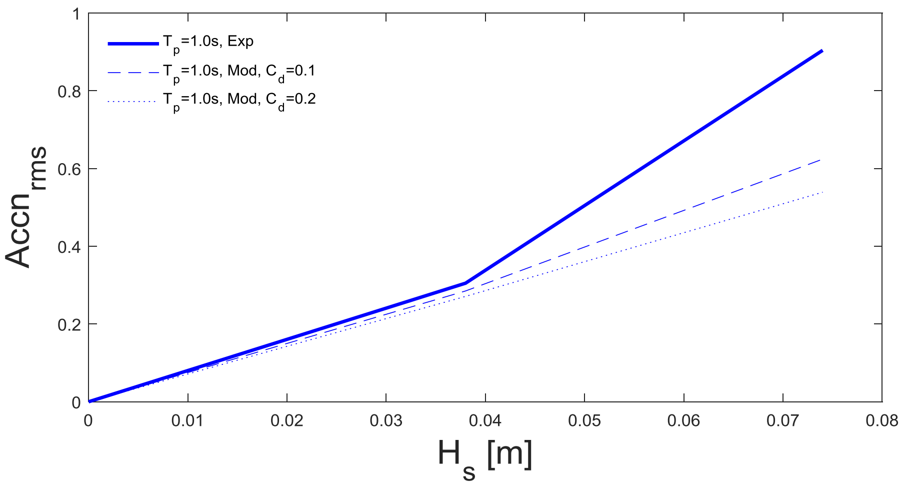




WEC power : mech, radiated



WEC accn rms in large waves with no PTO



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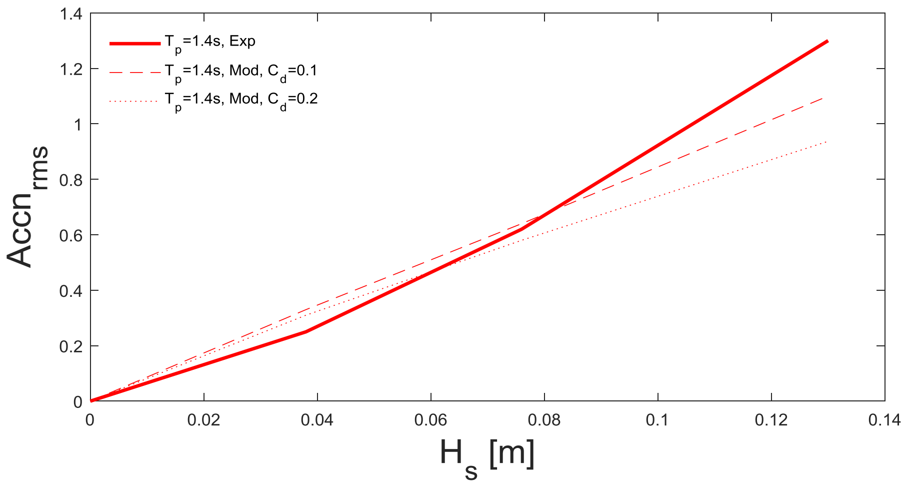

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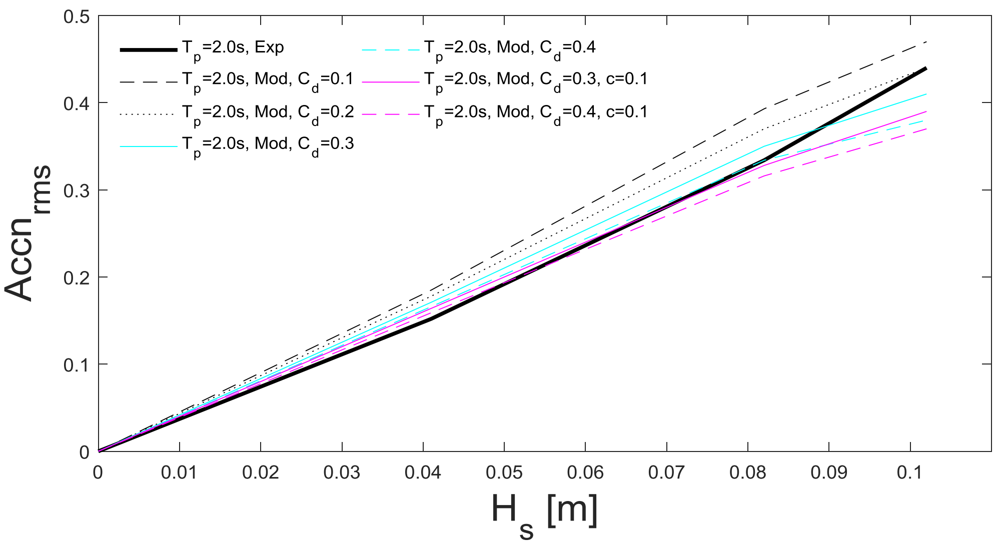

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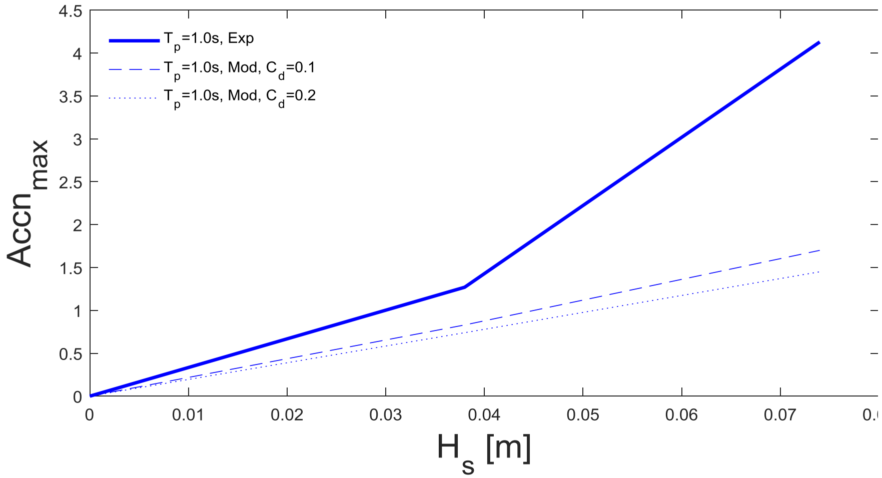
WEC accn max in large waves with no PTO



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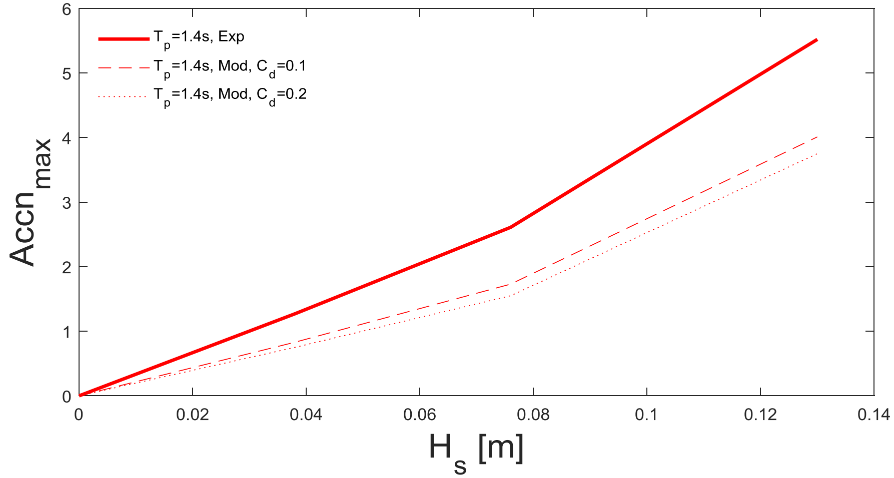

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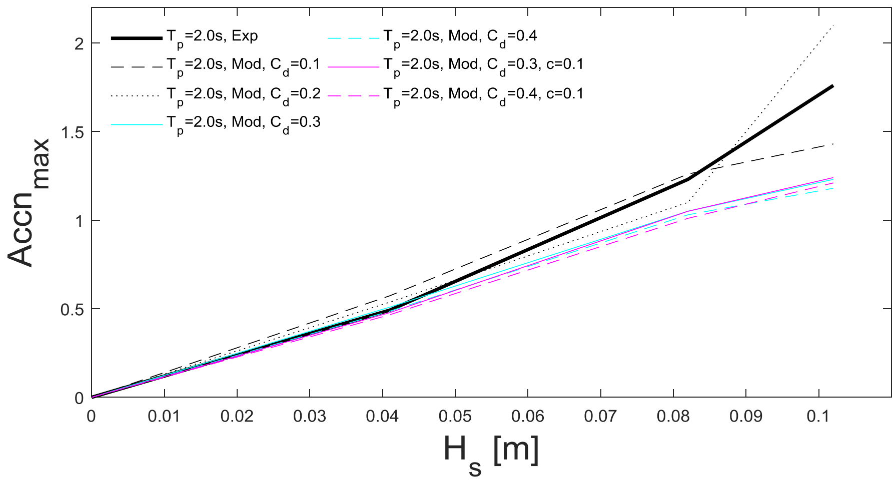

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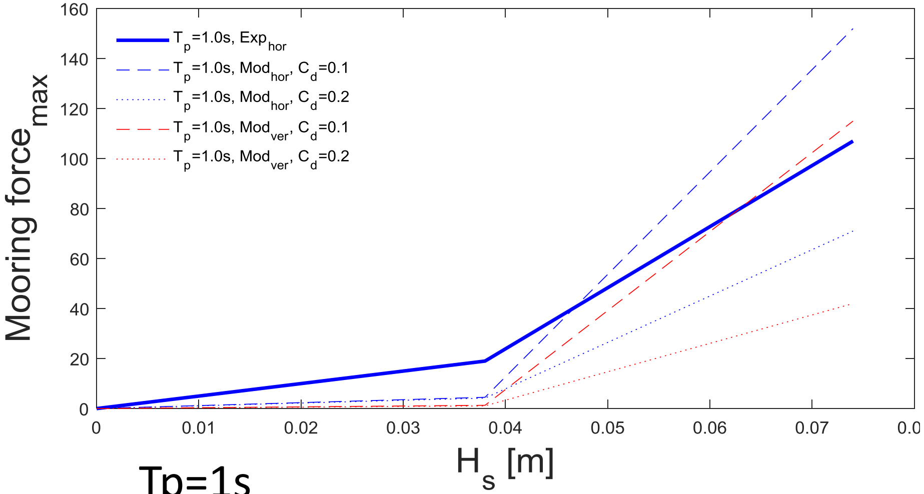
WEC in large waves mooring force max



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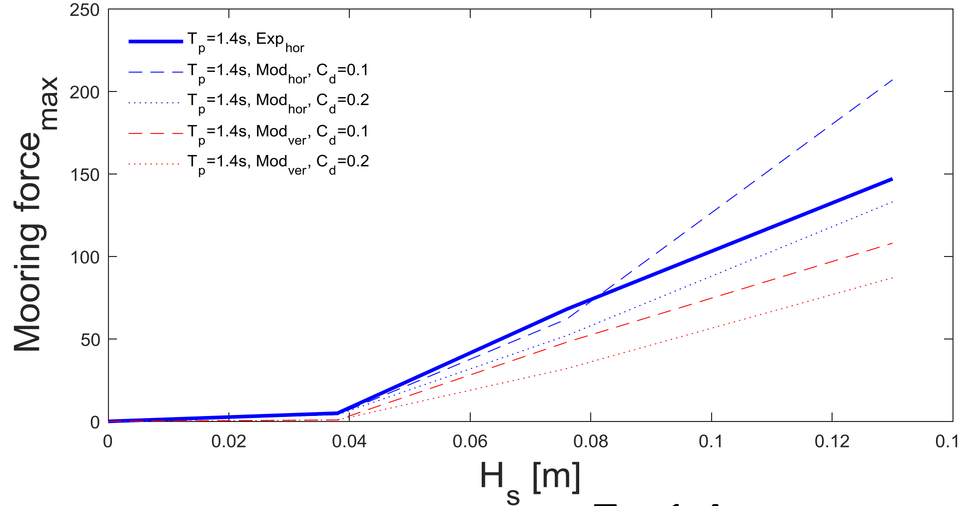

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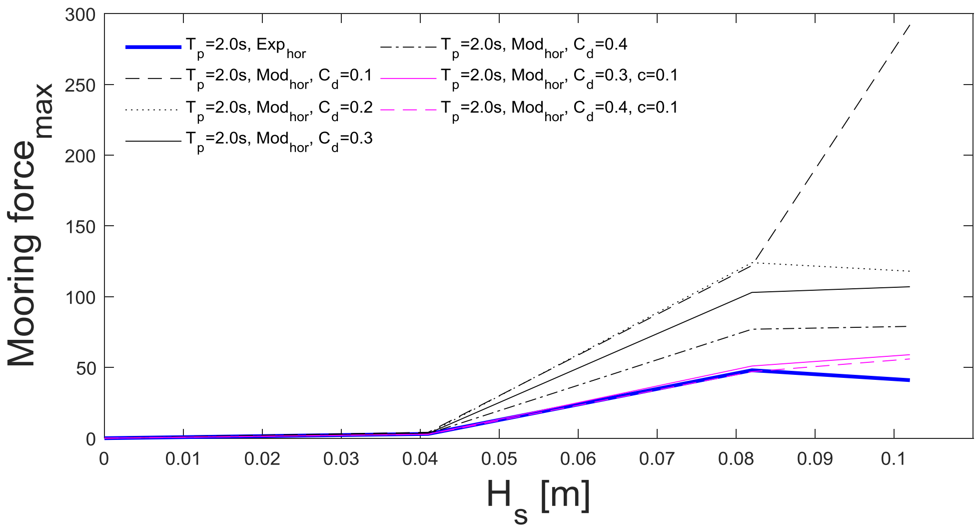

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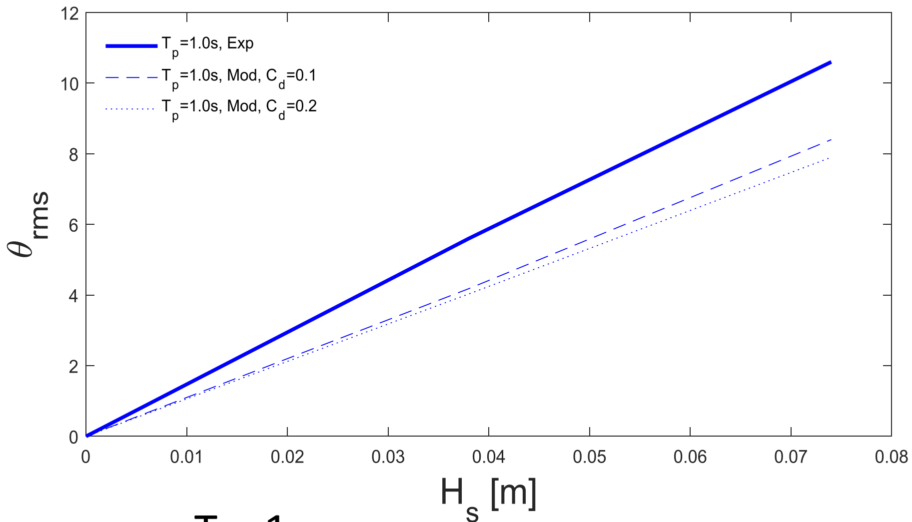
WEC in large waves angles rms



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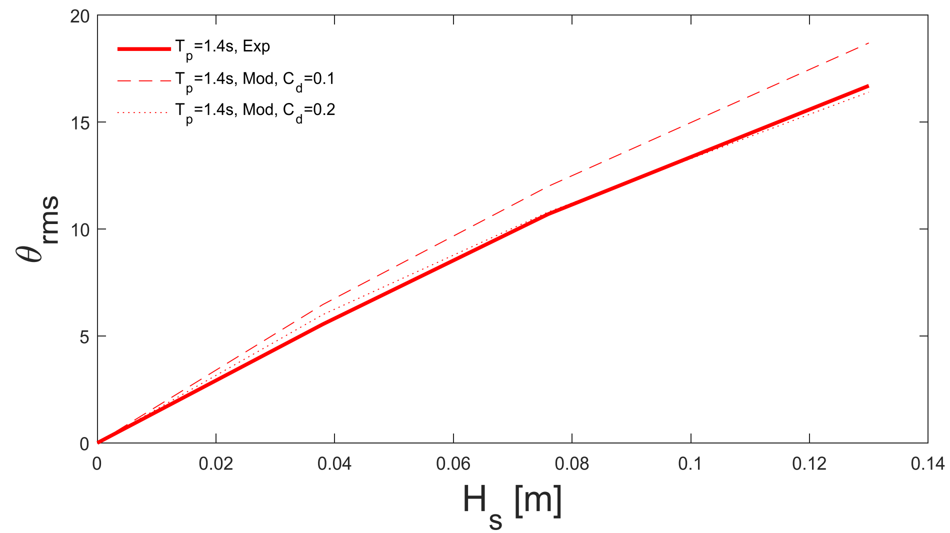

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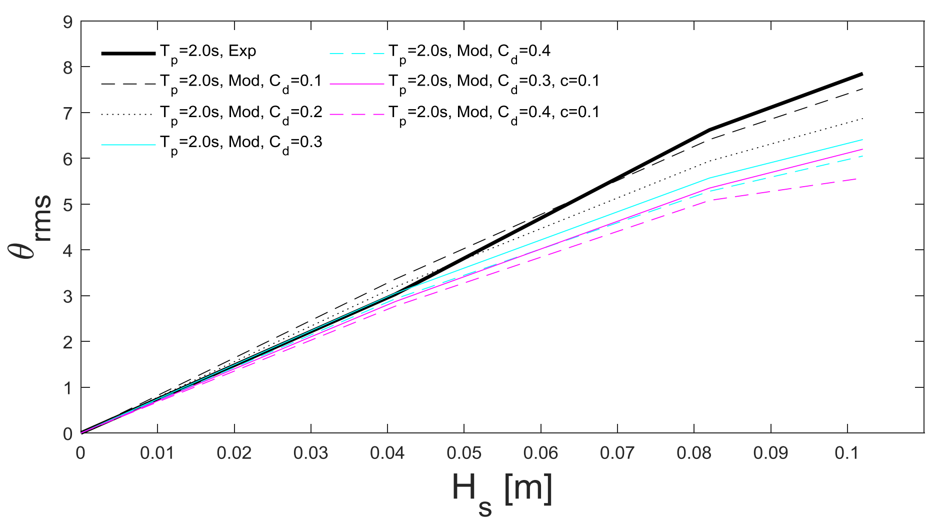

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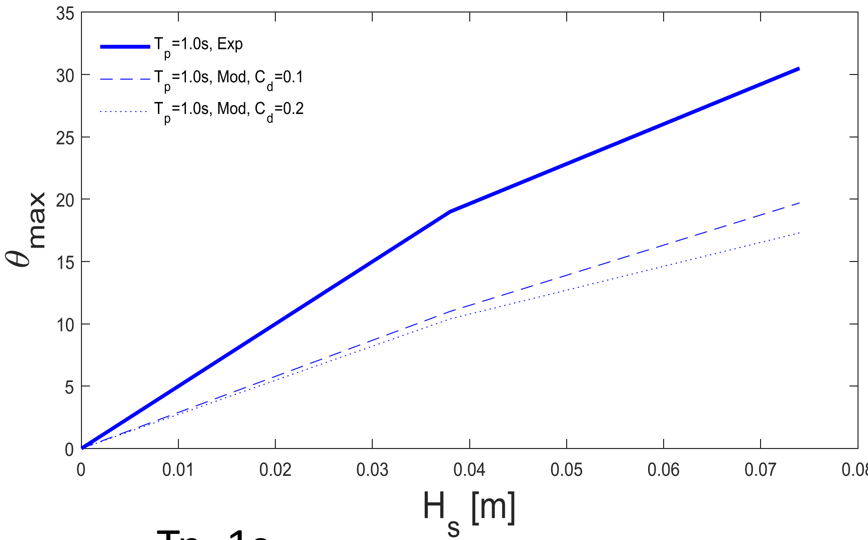
WEC in large waves angles max



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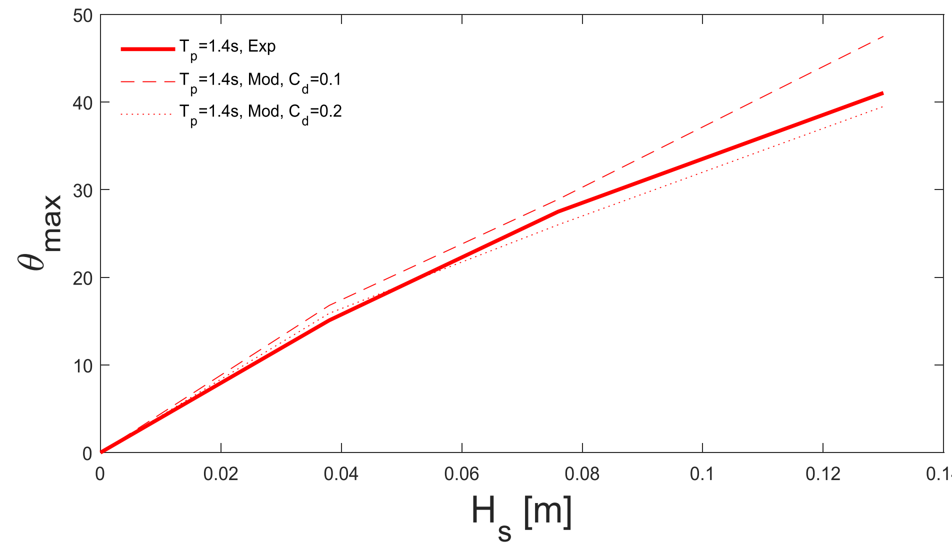

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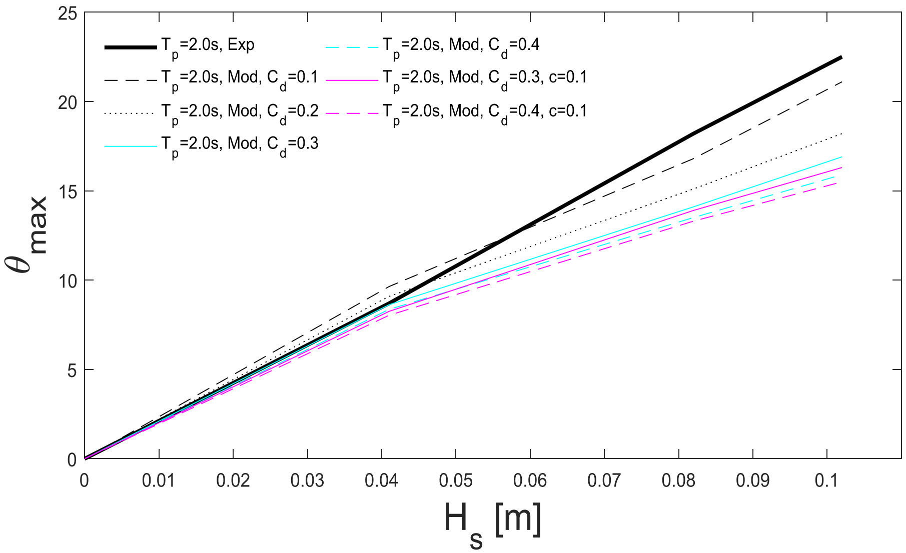

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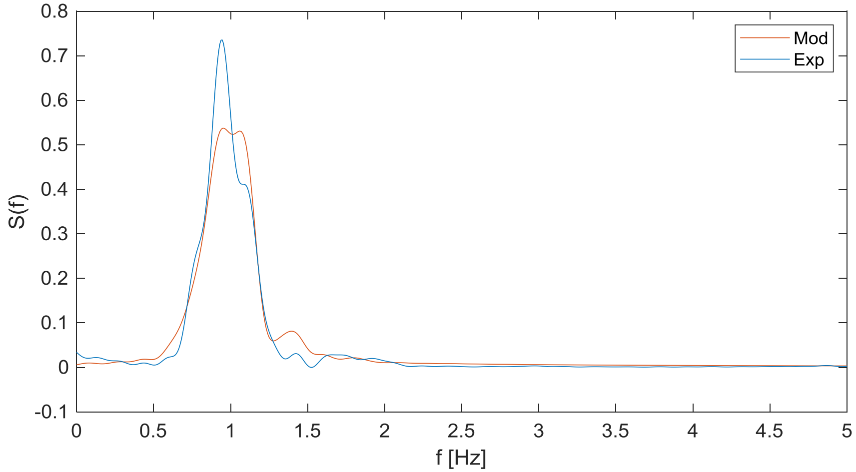
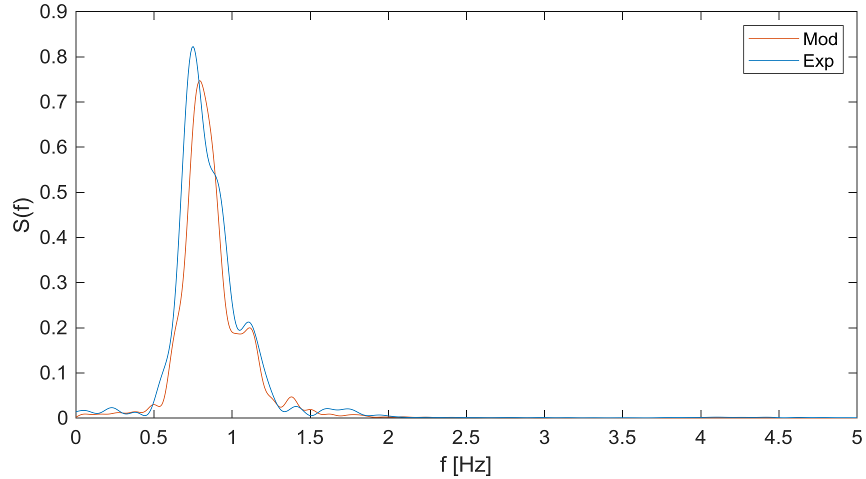
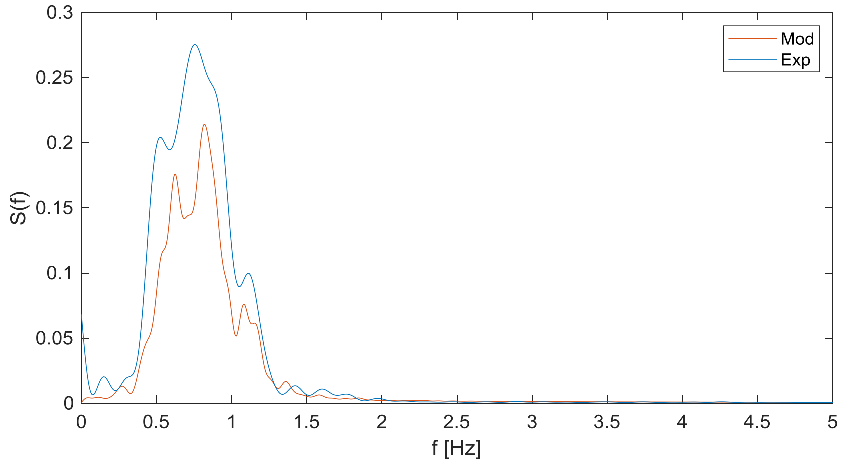
Relative angle spectra (operational)



Tp=1s



Tp=1.4s



Tp=2s

So far

- Wave power platforms generate similar to or more than offshore wind in some locations
- Linear model predicts power well for multi-float WECs in small – intermediate waves
- Rms angles and accn reasonably predicted even in large waves with realistic Cd
- Mooring force, mean and max underestimated in large waves

- WEC power capture potentially commercial

- Control is being developed (QMU London)
- Hydraulic PTO being designed (IST, Lisbon)
- Moorings to be improved

Control

- Latching – impedance matching - is well known
- Linear non-causal optimal controller more sophisticated but needs accurate and efficient model available for M4 as linear system
- future definition of the incoming waves is assumed to be available from other prediction techniques, e.g. AR

Zhijing Liao, Nian Gai, Peter Stansby, Guang Li
Liao, Gai and Li from Queen Mary University London

Convert to Euler Lagrange equation (3 float)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q \qquad q = [x_0\ z_0\ \theta_1\ \theta_2]^\top$$

where the Lagrangian $L := T - V$, with T as the total kinetic energy

$$T = \sum_{i=1,2,4}\left[\frac{1}{2}m_i(\dot{x}_i^2 + \dot{z}_i^2) + \frac{1}{2}I_i\dot{\theta}_1^2\right] + \sum_{i=3,5,6}\left[\frac{1}{2}m_i(\dot{x}_i^2 + \dot{z}_i^2) + \frac{1}{2}I_i\dot{\theta}_2^2\right] \tag{3}$$

and V as the total potential energy:

$$V = \sum_{i=1}^{6} m_i g z_i \tag{4}$$

Q is the generalized force acting on the system:

$$Q = f_{b,q} + f_{w,q} + f_{moor,q} + f_{drag,q} + f_{pto,q} \tag{5}$$

$$f_{w,q} = f_{e,q} + f_{rd,q} + f_{rs,q}$$

$$M\ddot{q}(t) = f_{e,q}(t) + f_{rd,q}(t) + f_{rs,q}(t) + f_{pto,q}(t)$$

$$M = \begin{bmatrix} \sum_{i=1}^{6} m_i & 0 & -m_1v_1-m_2v_2-m_4v_4 & -m_3v_3-m_5v_5-m_6v_6 \\ 0 & \sum_{i=1}^{6} m_i & m_1h_1+m_2h_2+m_4h_4 & -m_3h_3-m_5h_5-m_6h_6 \\ -m_1v_1-m_2v_2-m_4v_4 & m_1h_1+m_2h_2+m_4h_4 & \sum_{i=1,2,4}(I_i+m_i(h_i^2+v_i^2)) & 0 \\ -m_3v_3-m_5v_5-m_6v_6 & -m_3h_3-m_5h_5-m_6h_6 & 0 & \sum_{i=3,5,6}(I_i+m_i(h_i^2+v_i^2)) \end{bmatrix}$$

$$f_{e,q}(t) = \begin{bmatrix} f_{e,1,1} + f_{e,2,1} + f_{e,3,1} \\ f_{e,1,3} + f_{e,2,3} + f_{e,3,3} \\ f_{e,1,5} + f_{e,2,5} - f_{e,1,1}v_1 - f_{e,2,1}v_2 + f_{e,1,3}h_1 + f_{e,2,3}h_2 \\ f_{e,3,5} - f_{e,3,1}v_3 - f_{e,3,3}h_3 \end{bmatrix}$$

$$f_{rd,q}(t) = \begin{bmatrix} f_{rd,1,1} + f_{rd,2,1} + f_{rd,3,1} \\ f_{rd,1,3} + f_{rd,2,3} + f_{rd,3,3} \\ f_{rd,1,5} + f_{rd,2,5} - f_{rd,1,1}v_1 - f_{rd,2,1}v_2 + f_{rd,1,3}h_1 + f_{rd,2,3}h_2 \\ f_{rd,3,5} - f_{rd,3,1}v_3 - f_{rd,3,3}h_3 \end{bmatrix}$$

$$f_{rs,q}(t) = \begin{bmatrix} f_{rs,1,1} + f_{rs,2,1} + f_{rs,3,1} \\ f_{rs,1,3} + f_{rs,2,3} + f_{rs,3,3} \\ f_{rs,1,5} + f_{rs,2,5} - f_{rs,1,1}v_1 - f_{rs,2,1}v_2 + f_{rs,1,3}h_1 + f_{rs,2,3}h_2 \\ f_{rs,3,5} - f_{rs,3,1}v_3 - f_{rs,3,3}h_3 \end{bmatrix}$$

Cummins equation

$$f_{e,i,j}(t) = \sum_{n=1}^{200} H(n) F_{ex}(n, 6(i-1)+j) \cos(\phi(n, 6(i-1)+j) + \phi_{ran}(n))$$

$$L_{mn}(t) = \frac{2}{\pi} \int_0^{\infty} B_{mn}(\omega) \cos(\omega t) d\omega$$

float i in mode j

$$f_{rd,i,j}(t) = \sum_{n=1}^{3} \dot{x}_n * L_{6(i-1)+j,6(n-1)+1} + \sum_{n=1}^{3} \dot{z}_n * L_{6(i-1)+j,6(n-1)+3} + \sum_{n=1}^{2} \dot{\theta}_1 * L_{6(i-1)+j,6(n-1)+5} + \dot{\theta}_2 * L_{6(i-1)+j,6(n-1)+5|_{n=3}}$$

Convert to space state model using MATLAB

Generalised damping force

$$\dot{z}_s = A_s z_s + B_s \dot{q}(t)$$
$$f_{rd,q}(t) = C_s z_s + D_s \dot{q}(t)$$

space state representation

$$(M + m_\infty)\ddot{q}(t) + f_{rd,q}(t) + Kq(t) = f_{e,q}(t) + f_{pto,q}(t)$$
$$\dot{z}_s = A_s z_s + B_s \dot{q}(t)$$
$$f_{rd,q}(t) = C_s z_s + D_s \dot{q}(t) \quad (20)$$

Final space state equation

$$x := [q,\ \dot{q},\ \dot{z}_s]^\top$$

$$\dot{x} = Ax + B_w f_{e,q}(t) + B_u f_{pto,q}(t)$$
$$z = Cx + Du$$

where the system matrices are

$$A = \begin{bmatrix} 0_{4\times4} & I_{4\times4} & 0_{4\times n} \\ \frac{-K}{(M+m_\infty)} & \frac{-D_s}{(M+m_\infty)} & \frac{-C_s}{(M+m_\infty)} \\ 0_{n\times4} & B_s & A_s \end{bmatrix}$$
$$B_w = \begin{bmatrix} 0_{4\times4} \\ (M+m_\infty)^{-1} \\ 0_{n\times4} \end{bmatrix}$$
$$B_u = \begin{bmatrix} 0_{4\times1} \\ (M+m_\infty)^{-1}[0,0,1,-1]^\top \\ 0_{n\times1} \end{bmatrix}$$
$$C = \begin{bmatrix} 0_{1\times6} & 1 & -1 & 0_{1\times n} \end{bmatrix}$$
$$D = 0$$

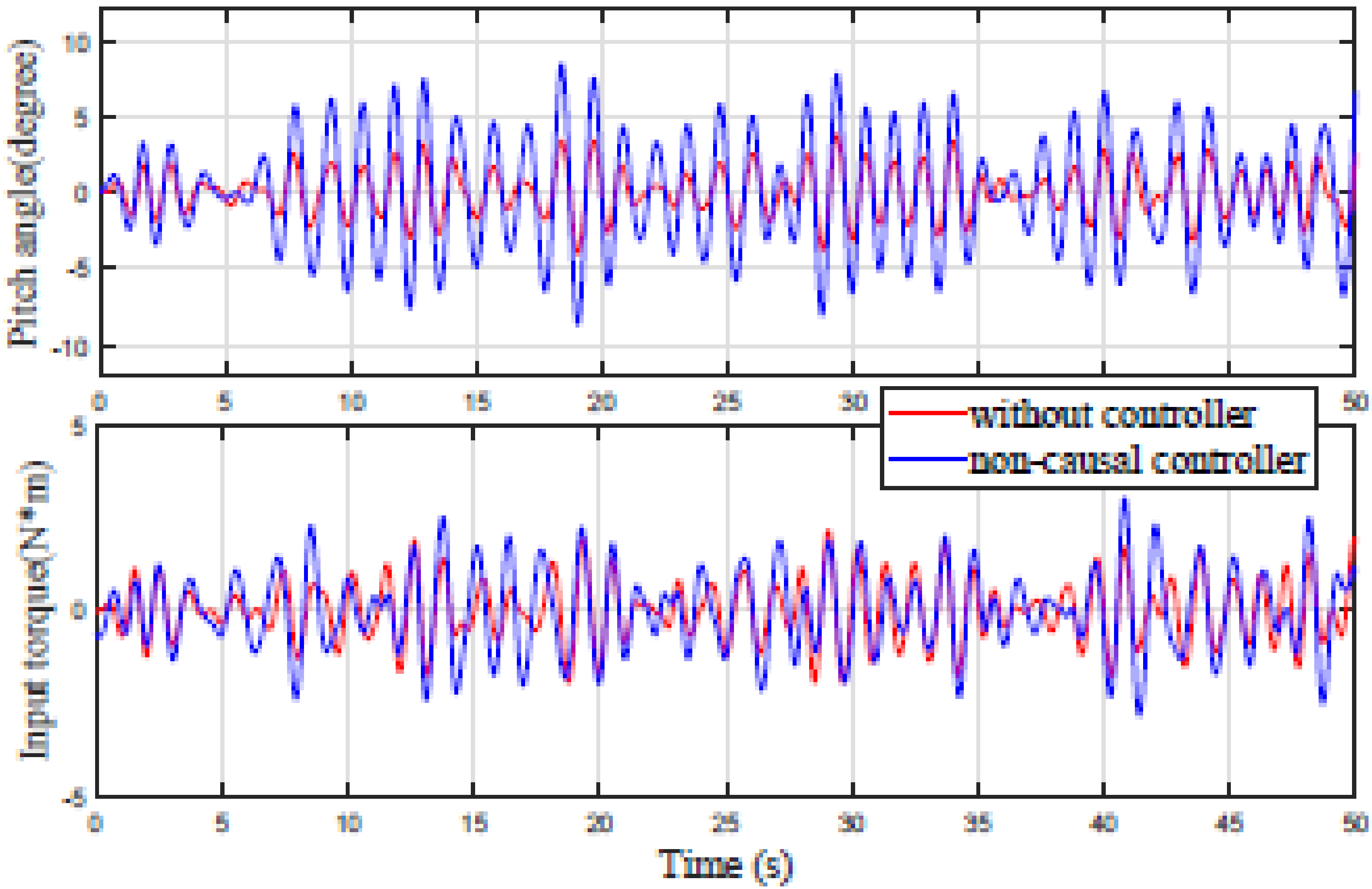
with $A \in \mathbb{R}^{136\times136}$

summary

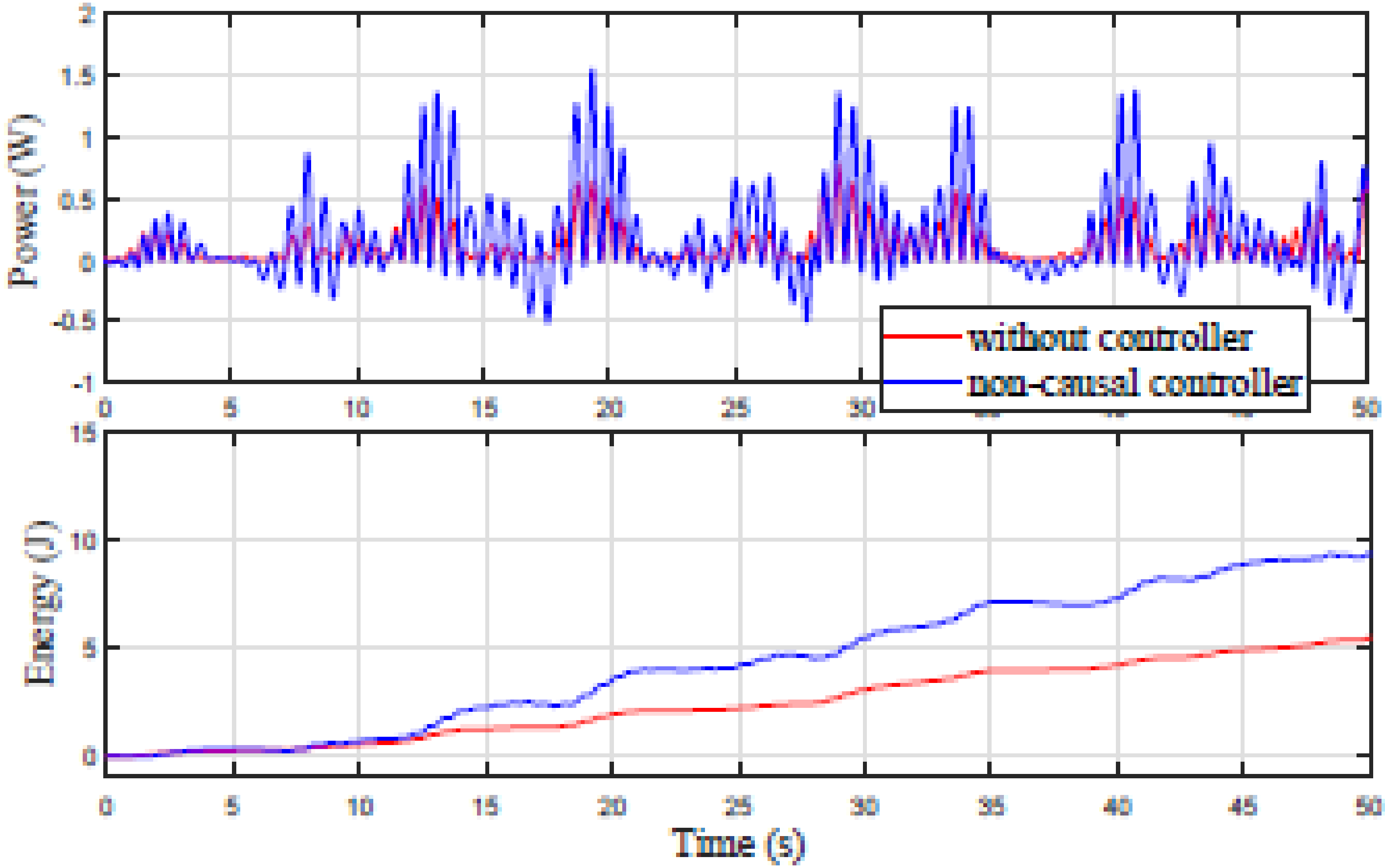
- Input – PTO control
- Output – relative pitch velocity
- Wave excitation is disturbance to system

- Cost function with three terms to maximise power, constrain motion, constrain torque
- Emphasis here on power

Example results Hs=0.04 m Tp=1.8 s



Power, energy



Capture width ratio

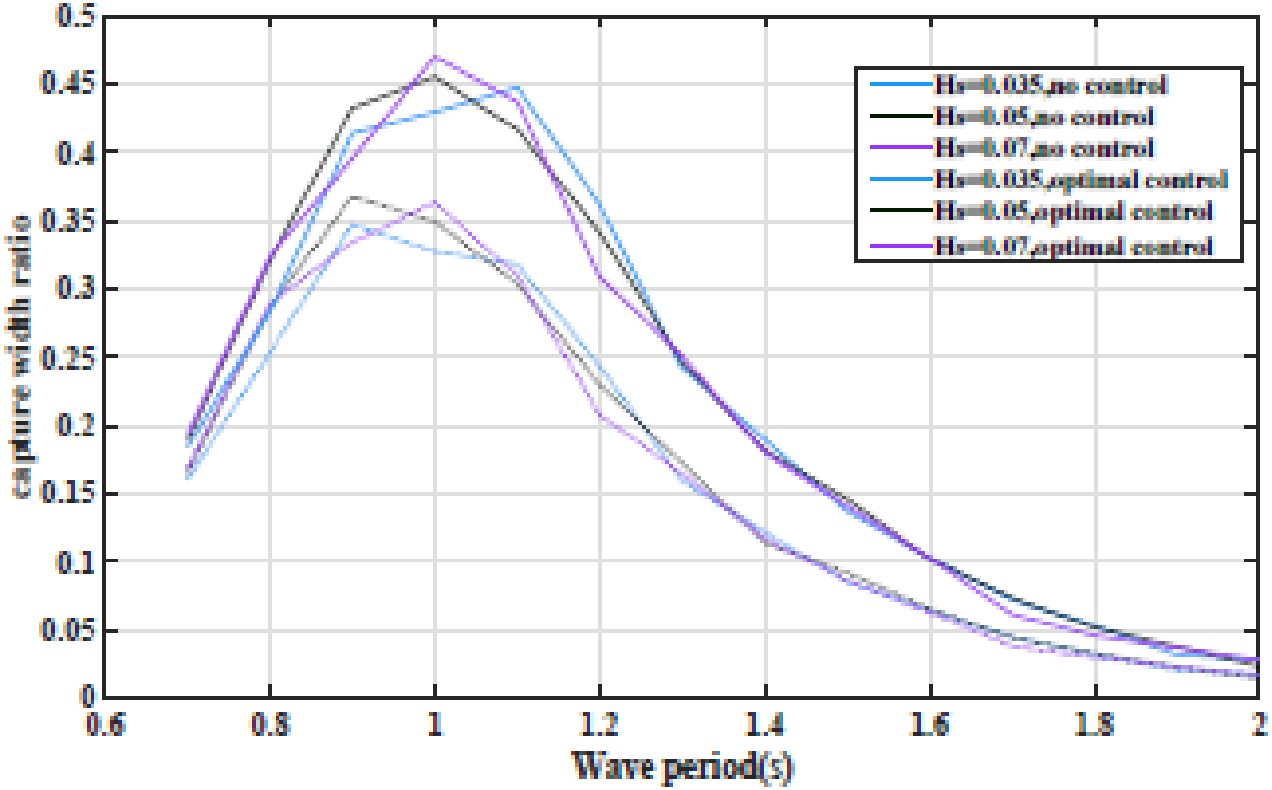


Fig. 6. Capture width ratio in different wave height and peak period, prediction horizon $2T_p = 3.6s$ ($n_p = 400$).

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RESULT

- CWR increased by 40% to 100%
- Combined with autoregressive (AR) model as wave excitation force predictor and Kalman filter with random walk wave model as wave excitation force estimator
- Gives energy close to exact wave force knowledge

Controller framework

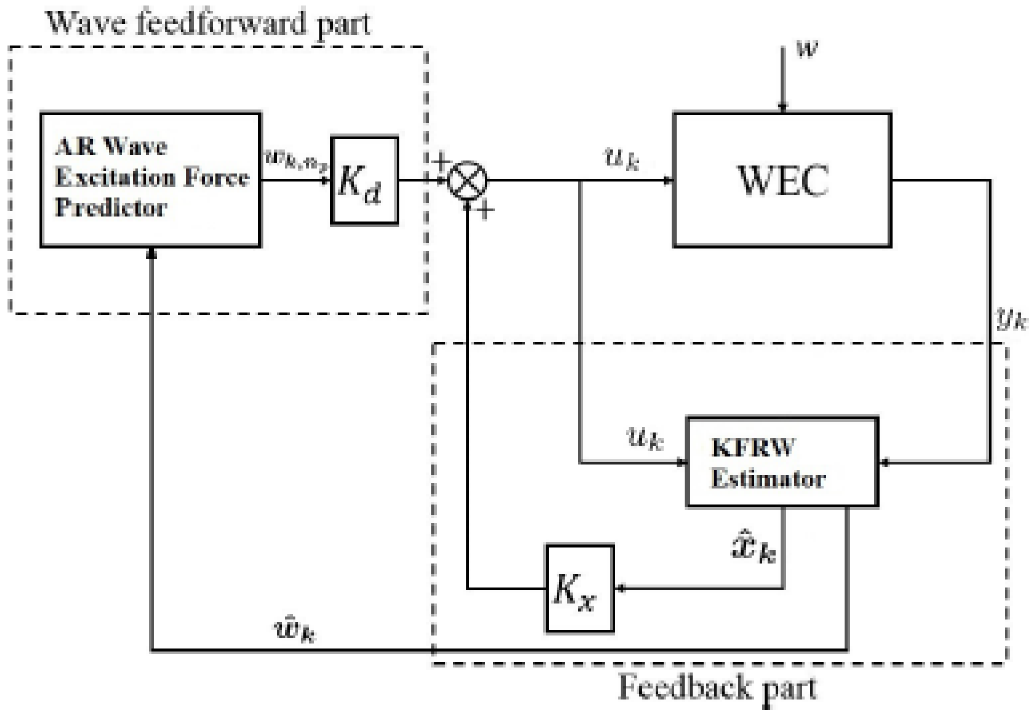

Figure 1: Complete linear non-causal optimal controller framework.

2-3 forward wave periods predicted

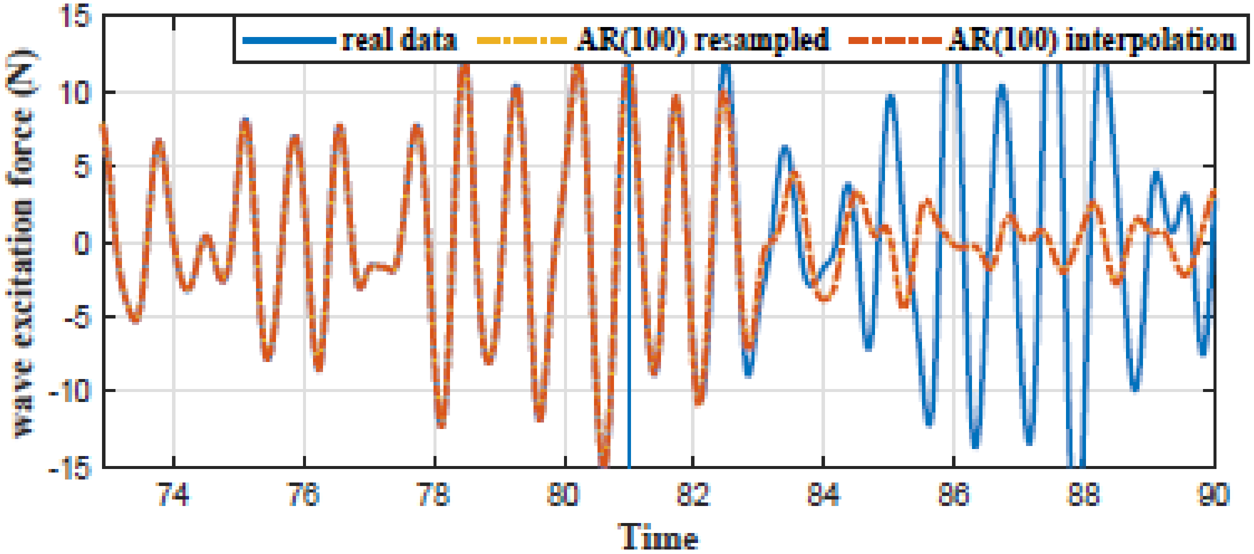

Figure 6: AR model with $p = 100$, $N = 8000$ re-sampled to predict wave excitation force with JONSWAP wave significant height $H_s = 0.04m$ and peak period $T_p = 1s$.

Energy capture

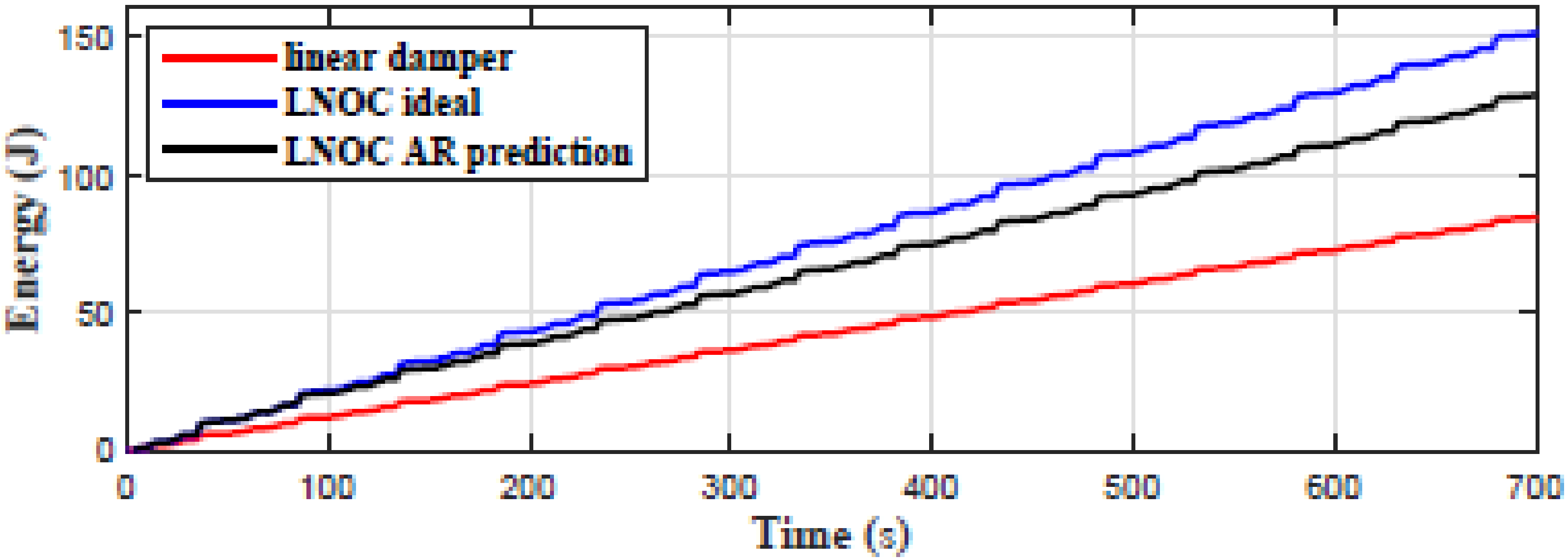

Figure 7: Energy captured, JONSWAP wave profile $H_s = 0.04m$, $T_p = 1.8s$.

CWR

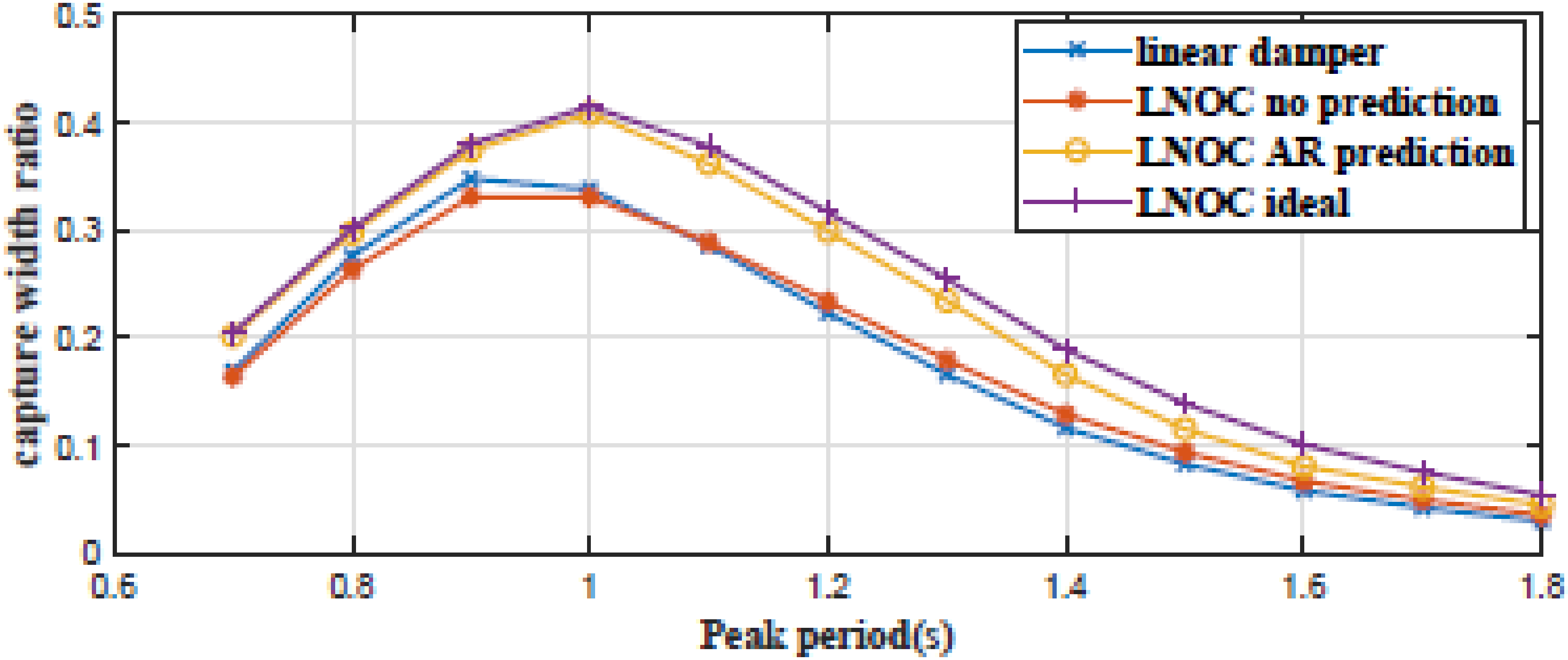

Figure 11: Capture width ratio, JONSWAP wave profile $H_s = 0.04m$, wave excitation force prediction horizon is $2 \times T_p$.

conclusions

- Linear model works well for motion and power
- Multi-degree of freedom control works well
- AR wave force prediction effective and efficient

- Need to improve and model mooring
- Need to design configuration to reduce peak to mean power for PTO
- Need to manufacture PTO
- Hybrid wind and wave has advantages for smoother combined supply