# Moment-based identification for wave energy systems

## Fundamentals and application

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# Introduction and motivation

## Motivation

The equations of motion of wave energy devices, in the LTI case, can be expressed in terms of **Cummins' equation**.

> **Integro-differential Volterra equation**, of the convolution class.

This convolution term represents fluid memory effects associated with the dynamics of the **radiation forces**.

**Inconvenient for simulation and analysis and design of control systems!**

> How can we solve this issue and find a more tractable system?

$$\dot{x}(t) = v(t),$$
$$\dot{v}(t) = \frac{1}{m + \mu_\infty}\left[ - s_h x(t) - \underbrace{\int_0^{+\infty} \zeta(\tau) v(t-\tau) d\tau}_{y_r(t)} + \mathcal{F}_e(t)\right].$$

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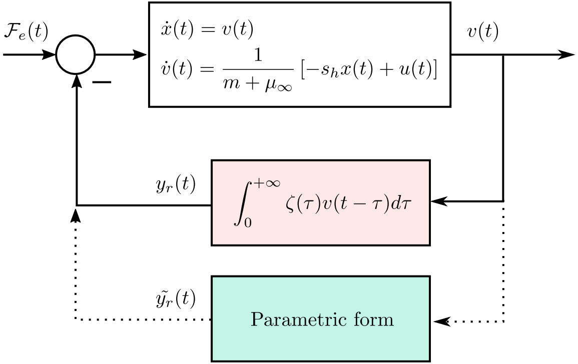


$$\|y_r(t) - \tilde{y_r}(t)\| < \epsilon$$

# Force-to-velocity parameterisation

$$\dot{x}(t) = v(t),$$

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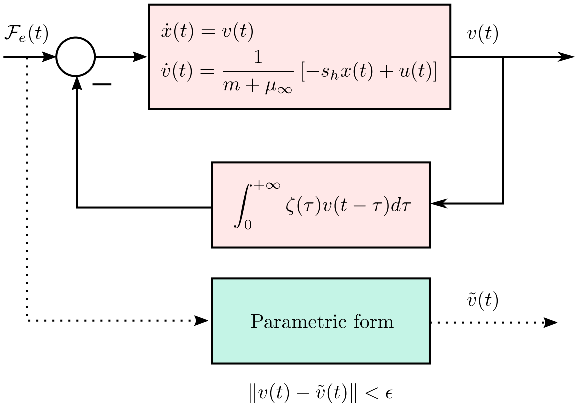


These non-parametric models are usually obtained with BEM solvers

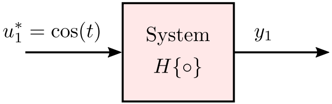
- WAMIT
- NEMOH

**BEM solvers are computationally efficient, but they can only characterise the steady-state characteristics of the WEC.**

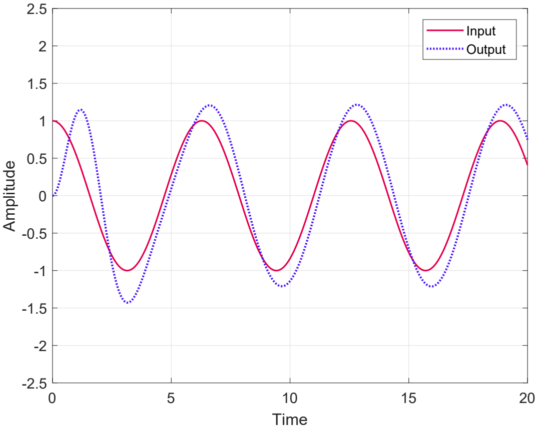
Non-parametric data to identify:

Finite number of points in **frequency-domain** $\longrightarrow$ frequency response!

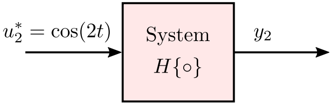
# Frequency response fundamentals

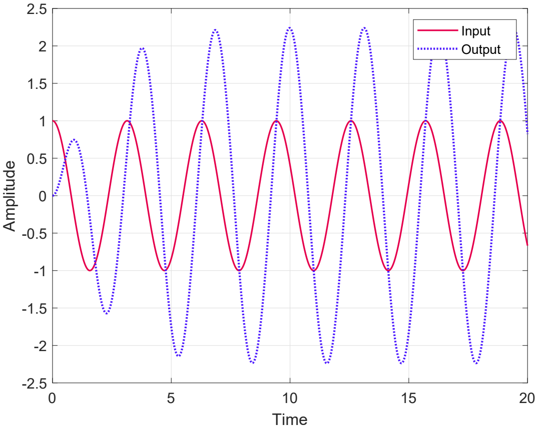


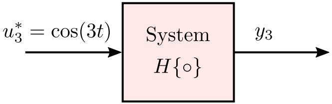
$u_1^* = \cos(t)$
System
$H\{\circ\}$
$y_1$

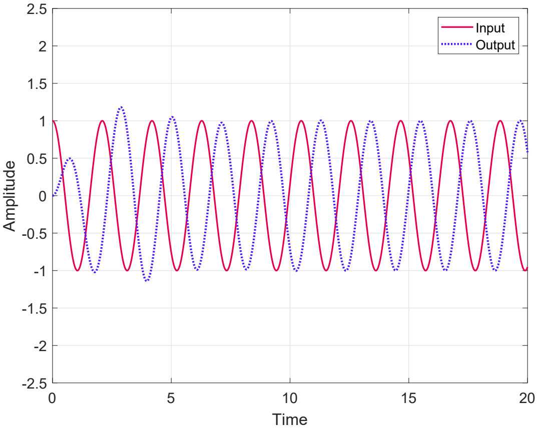



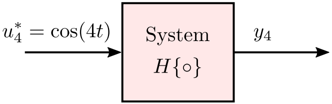
Input
Output
Amplitude
Time
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
5
10
15
20

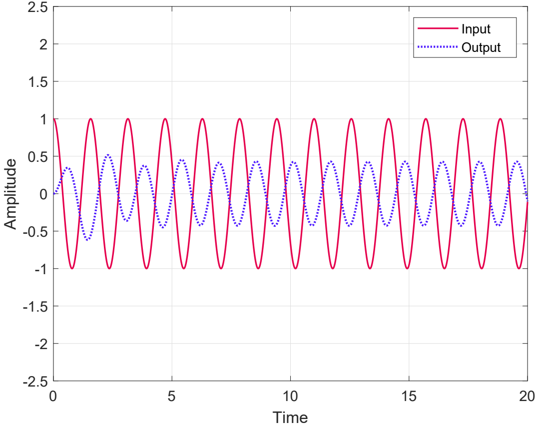

$u_2^* = \cos(2t)$
System
$H\{\circ\}$
$y_2$

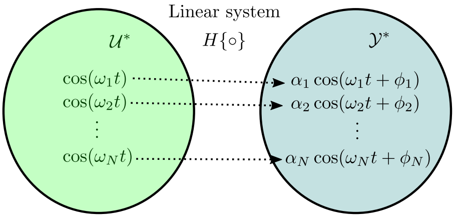

2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
Amplitude
0
5
10
15
20
Time
Input
Output

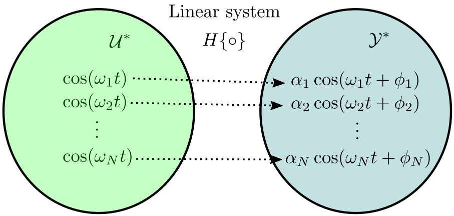

$u_3^* = \cos(3t)$
System
$H\{\circ\}$
$y_3$


Input
Output
Amplitude
Time
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
5
10
15
20


$u_4^* = \cos(4t)$
System
$H\{\circ\}$
$y_4$


Input
Output
Amplitude
Time
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
5
10
15
20


Linear system
$H\{\circ\}$
$\mathcal{U}^*$
$\mathcal{Y}^*$
$\cos(\omega_1 t)$
$\cos(\omega_2 t)$
$\vdots$
$\cos(\omega_N t)$
$\alpha_1 \cos(\omega_1 t + \phi_1)$
$\alpha_2 \cos(\omega_2 t + \phi_2)$
$\vdots$
$\alpha_N \cos(\omega_N t + \phi_N)$


Linear system
$H\{\circ\}$
$\mathcal{U}^*$
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$\cos(\omega_1 t)$
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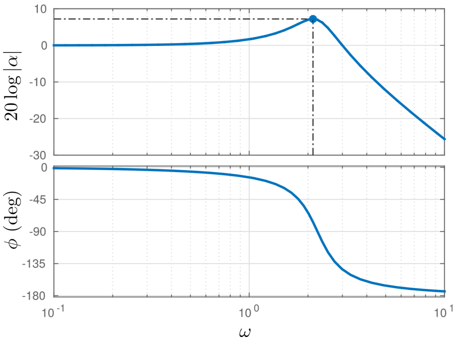
$\cos(\omega_N t)$
$\alpha_1 \cos(\omega_1 t + \phi_1)$
$\alpha_2 \cos(\omega_2 t + \phi_2)$
$\vdots$
$\alpha_N \cos(\omega_N t + \phi_N)$


$20 \log |\alpha|$
10
0
-10
-20
-30
$\phi$ (deg)
0
-45
-90
-135
-180
$10^{-1}$
$10^{0}$
$10^{1}$
$\omega$

# Moment-based identification

**Fundamental questions:**

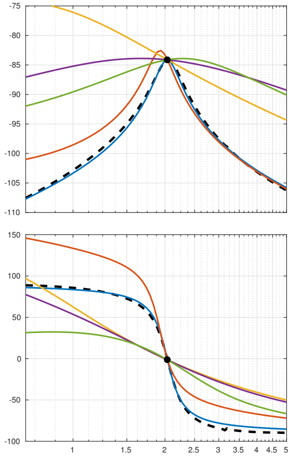
Given a non-parametric set of data in **frequency-domain**, how do we select a family of parametric models that can represent our data?

Once this family is decided, how to we choose the best model inside this set?

**An ideal parametric modeling technique should:**

- Preserve underlying physical characteristics.
- Preserve a precise steady-state description for key important frequencies.
- Minimise approximation error in a particular frequency range of interest.

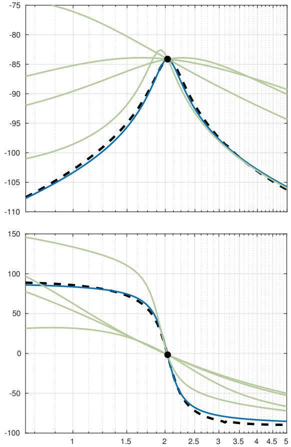
We propose a moment-based approach:




**Moment-based family of models:**

- LTI models of order $\nu$ that preserve an **exact** steady-state response for a user-defined set of frequencies $\mathscr{F} = \{\omega_p\}_{i=1}^{p}$.
- Model order $\nu = 2p$ (twice the number of frequencies to interpolate).
- Parameterised in state-space form $(A, B, C, 0)$.

## We propose a moment-based approach:




**Moment-based optimal model:**

- Preserve internal stability.
- Interpolation in the frequency set $\mathscr{F}$.
- Optimal approximation in Euclidean sense for a *user-defined frequency range*.

## Application case: OPT-device

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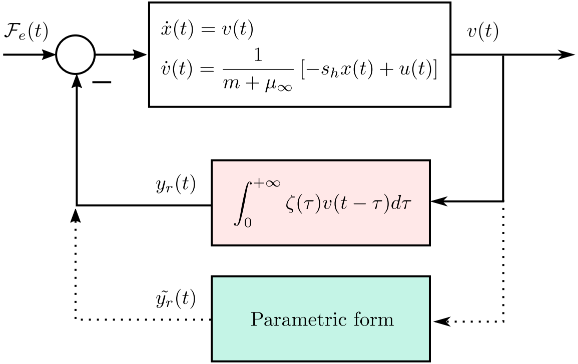
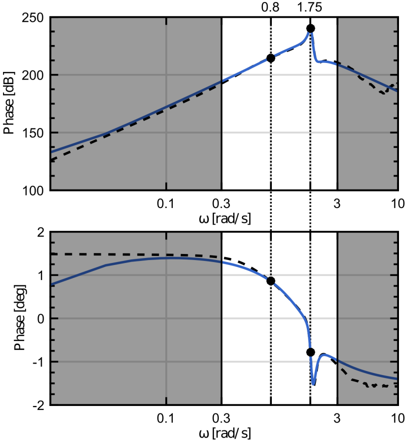
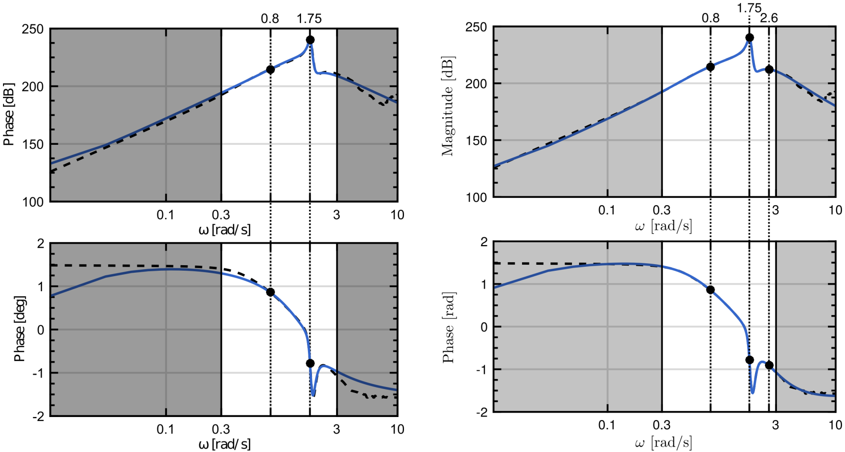



$$\|y_r(t) - \tilde{y_r}(t)\| < \epsilon$$




Target frequency-domain data in (dashed-black). Parametric approximation in (solid-blue).

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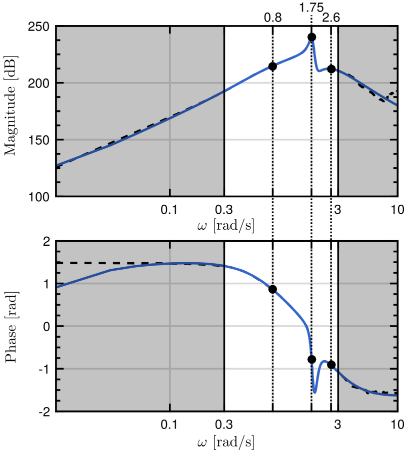
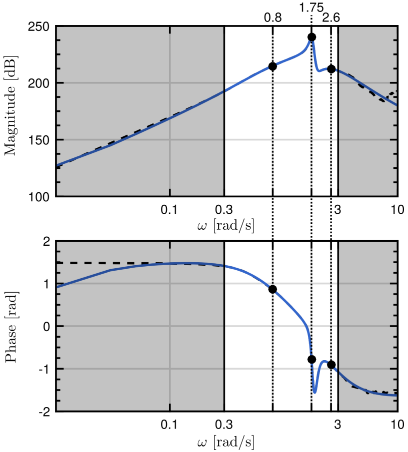
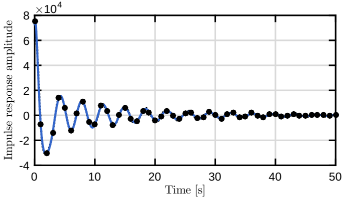



Target non-parametric data in black. Parametric approximation in blue.

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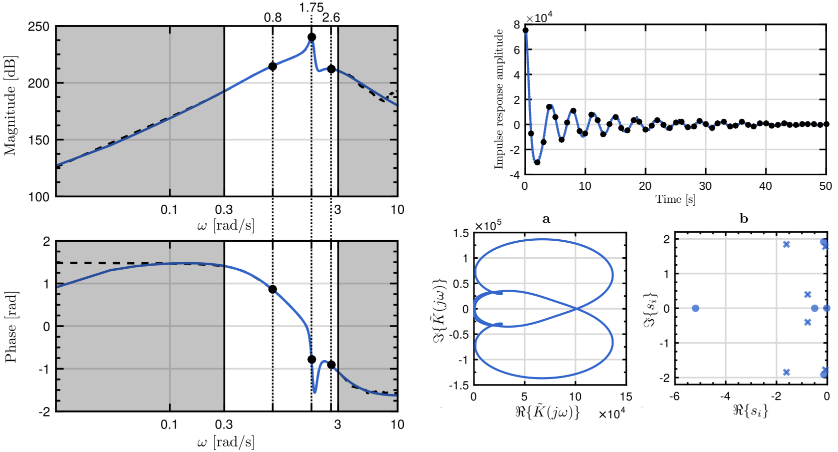


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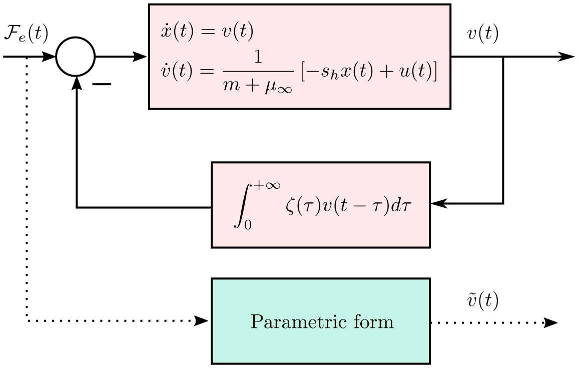
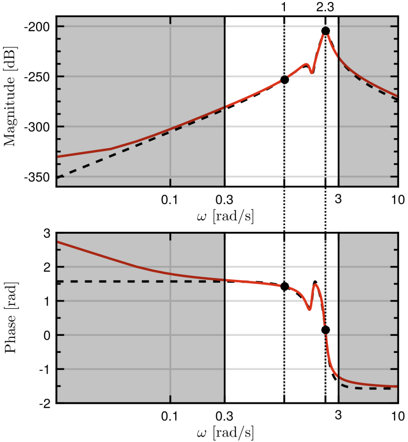


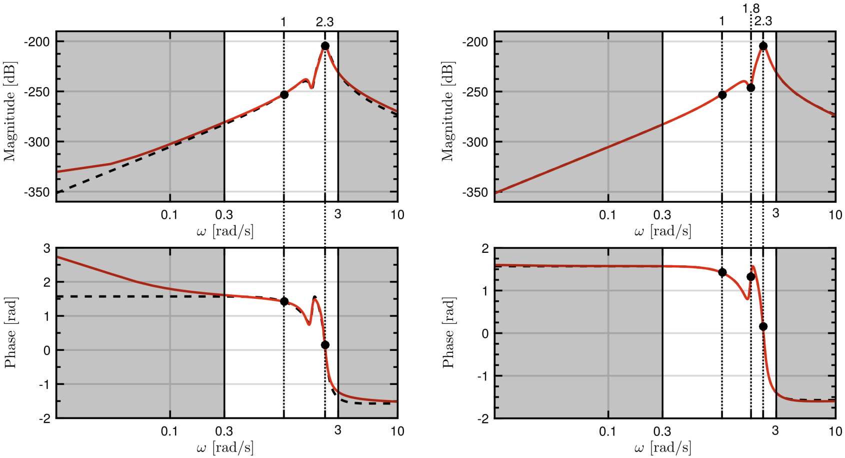

$$\|v(t) - \tilde{v}(t)\| < \epsilon$$

# Force-to-velocity



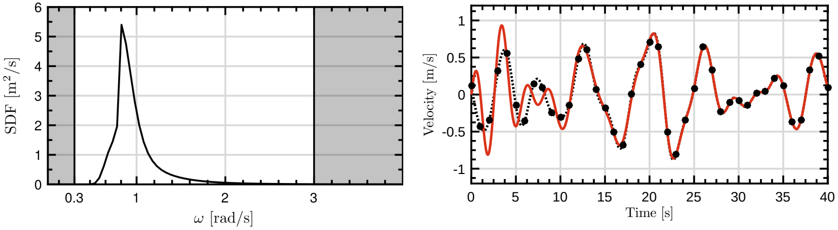

Target frequency-domain data in (dashed-black). Parametric approximation in (solid-red).

## Force-to-velocity



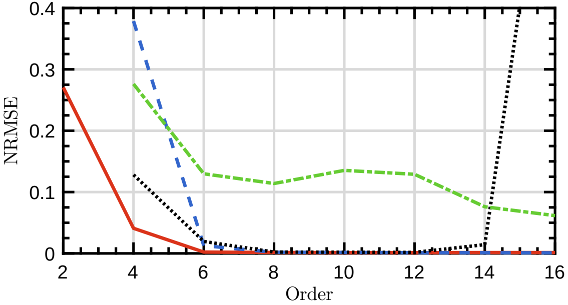

Target frequency-domain data in (dashed-black). Parametric approximation in (solid-red).

# Force-to-velocity




Target non-parametric data in black. Parametric approximation in red.

**Force-to-velocity response comparison:**




**Solid-red** Force-to-velocity based on moments.

**Dashed-blue** Cummins' equation + radiation force model based on moments.

**Dotted-black** Cummins' equation + radiation force model based on *invfreqs* function (NTNU).

**Dashed-green** Cummins' equation + radiation force model based on Prony's method.

# Conclusions

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- We have developed a moment-matching-based identification algorithm that can be applied to obtain a parametric form of both the **force-to-motion** and the **radiation impulse response** dynamics from raw frequency-domain data.
- The strategy provides a parametric model that matches *exactly* the behaviour of the device at key frequencies that can be selected by the user.
- The strategy allows for the selection of a **frequency range** to perform the approximation.
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**F**inite-**O**rder **A**pproximation by **M**oment-**M**atching toolbox $\longrightarrow$ **FOAMM**

- Nicolás Faedo, Yerai Peña-Sanchez, and John V Ringwood. "Finite-order hydrodynamic model determination for wave energy applications using moment-matching". In: *Ocean Engineering* 163 (2018), pp. 251–263.
- Nicolás Faedo, Yerai Peña-Sanchez, and John V Ringwood. "Moment-Matching-Based Identification of Wave Energy Converters: the ISWEC Device". In: *IFAC-PapersOnLine* 51.29 (2018), pp. 189–194.
- Nicolás Faedo, Yerai Peña-Sanchez, and John V. Ringwood. "Parameterisation of Radiation Forces for a Multiple Degree-of-Freedom Wave Energy Converter Using Moment-Matching". In: *Proceedings of the 2019 Automatic Control Conference, Philadelphia*. 2019.
- Nicolás Faedo, Yerai Peña-Sanchez, and John V Ringwood. "Passivity preserving moment-based finite-order hydrodynamic model identification for wave energy applications". In: *Advances in Renewable Energies Offshore: Proceedings of the 3rd International Conference on Renewable Energies Offshore (RENEW 2018), Lisbon*. 2018, p. 351.

Yerai Peña-Sanchez, Nicolás Faedo, and John V. Ringwood. "Hydrodynamic Model Fitting for Wave Energy Applications Using Moment-Matching: A Case Study". In: *The 28th International Ocean and Polar Engineering Conference*. International Society of Offshore and Polar Engineers. 2018.

Yerai Peña-Sanchez, Nicolás Faedo, and John V. Ringwood. "Moment-Based Parametric Identification of Arrays of Wave Energy Converters". In: *Proceedings of the 2019 Automatic Control Conference, Philadelphia*. 2019.

**Thanks!**

**Any questions?**

Centre for Ocean Energy Research